Superradiance of Degenerate Fermi Gases in a Cavity

Zhenhua Yu
Institute for Advanced Study
Tsinghua University

Collaborators: Yu Chen and Hui Zhai
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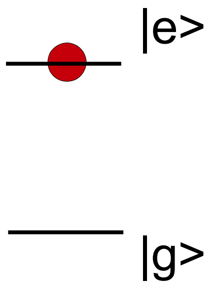
Cavity QED

*Discrete spectrum -> one or a few relevant cavity modes
*Possible strong coupling at ***SINGLE*** photon level

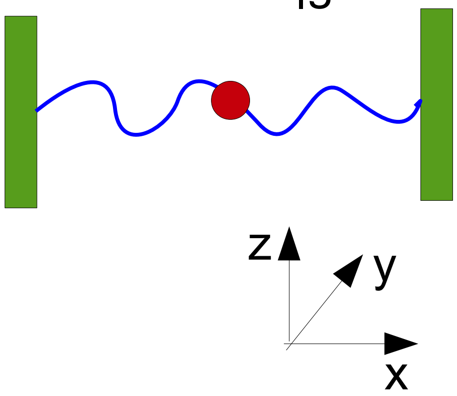
Electronic dipole coupling between single atoms and light field

$$h_{dip} = \boldsymbol{E} \cdot \boldsymbol{d}$$



With canonical quantization

$$h_{dip} = \sum_{\boldsymbol{k},\epsilon} g_{k,\epsilon} \left(i\, a_{\boldsymbol{k},\epsilon} e^{i \boldsymbol{k} \cdot \boldsymbol{r}} + h.c. \right)$$



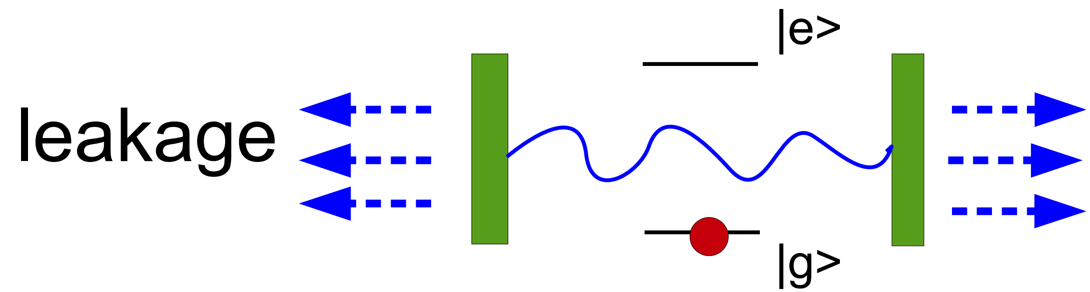
$$g_{k,\epsilon} = \sqrt{\frac{\omega_k}{2\Omega}} \boldsymbol{\epsilon} \cdot \boldsymbol{d} \sim \sqrt{\omega_k \frac{e^2}{a_0}} \sqrt{\frac{a_0^3}{\Omega}} \sim 10\, MHz$$

$e\, a_0$ $\quad 10^{14}\, Hz$ $\quad 1\, \mu m^3$

Strong coupling

Decay rate

$$\kappa \sim 1\, MHz$$

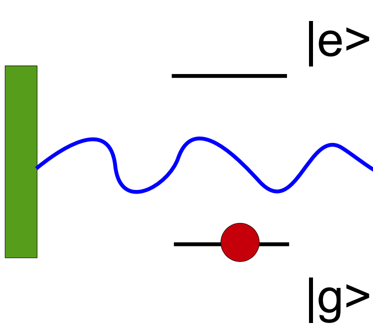


For a resonant cavity mode whose frequency is equal to the electronic excitation energy of the atom,

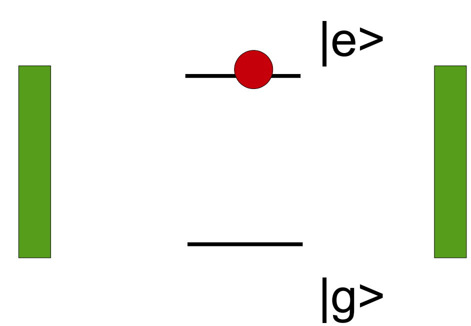
$$i\frac{\partial a_{\boldsymbol{k},\epsilon}}{\partial t} = -i\kappa a_{\boldsymbol{k},\epsilon} - ig_{\boldsymbol{k},\epsilon}\sigma_{-}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}$$

$$\sigma_{-} = |\mathrm{g}\rangle\langle\mathrm{e}|$$

$$i\frac{\partial}{\partial t}\sigma_{-} = i\,g_{\boldsymbol{k},\epsilon}a_{\boldsymbol{k},\epsilon}e^{i\boldsymbol{k}\cdot\boldsymbol{r}},$$



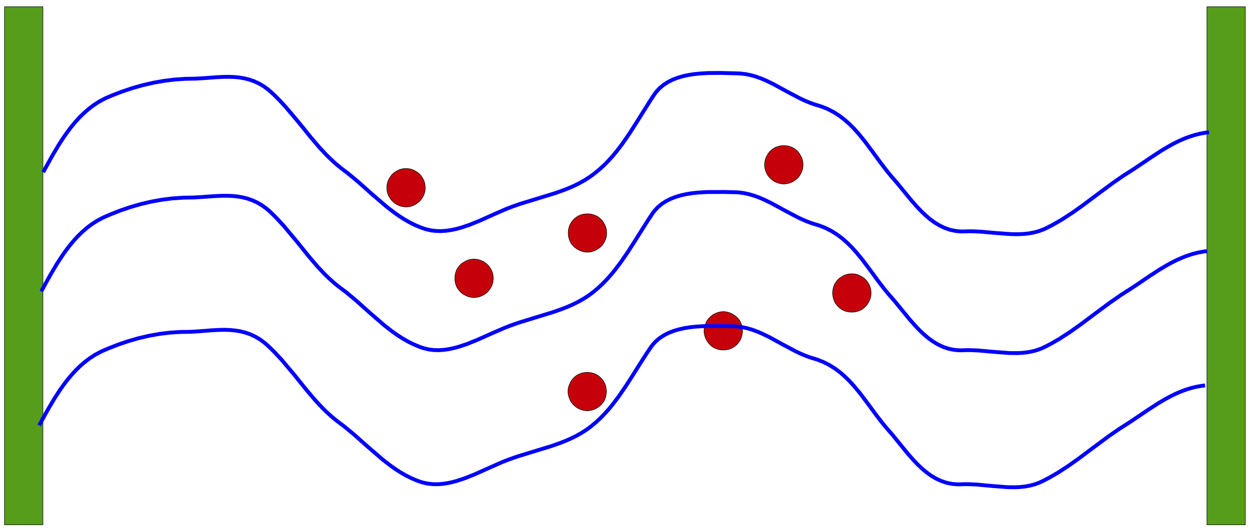
Ocsillation



$$t < 1/\kappa,\ \omega \sim g_{\boldsymbol{k},\epsilon}$$

What for many atoms?

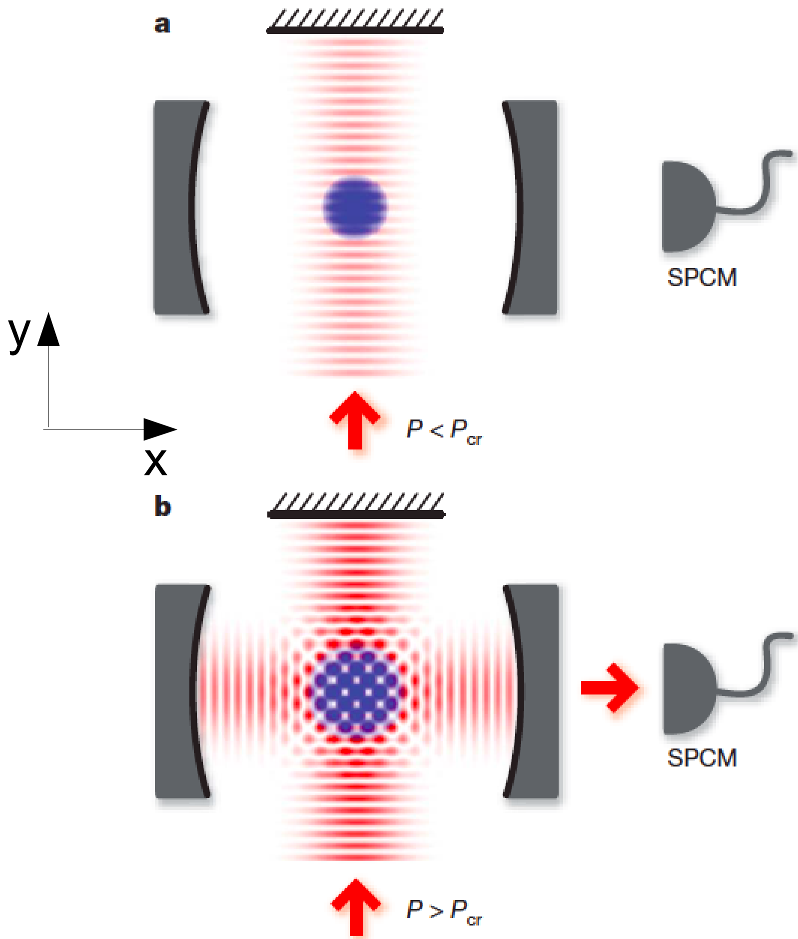
Correlations between atoms can be built by the mediated interactions via the same cavity field.

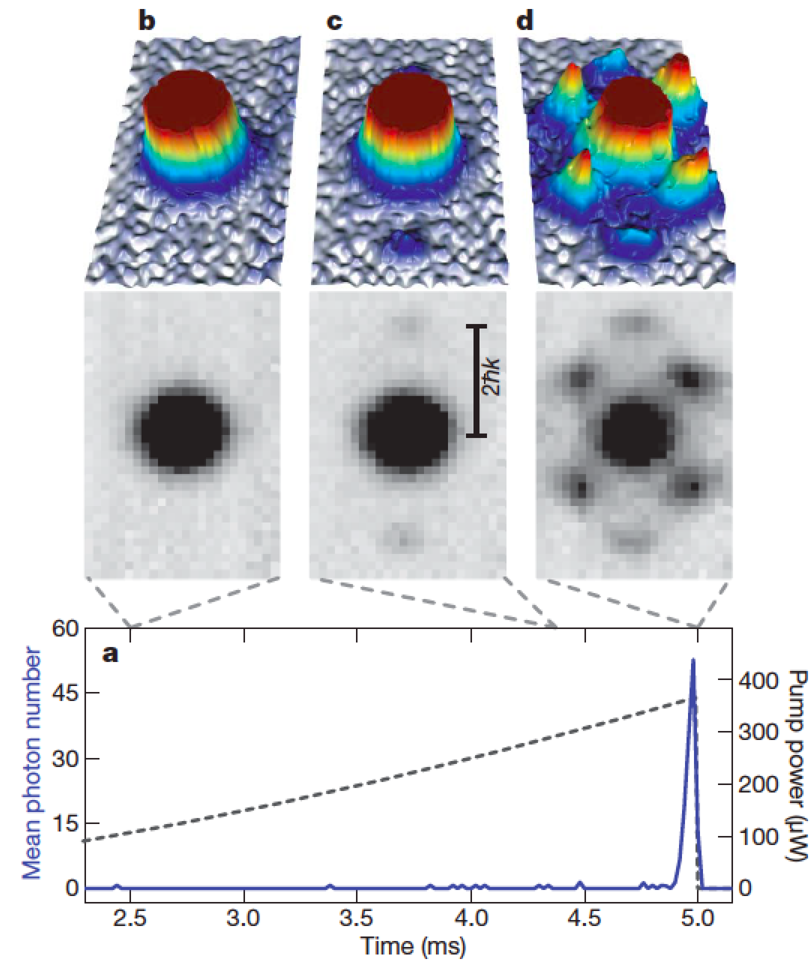


Realizing long range interactions ???

- Atoms in optical cavities
- Atoms with magnetic dipole moments (Dy)--small magnitude
- Rydberg atoms—limited lifetime
- Molecules with electric dipole moments

Superradiance in BEC with a single cavity mode





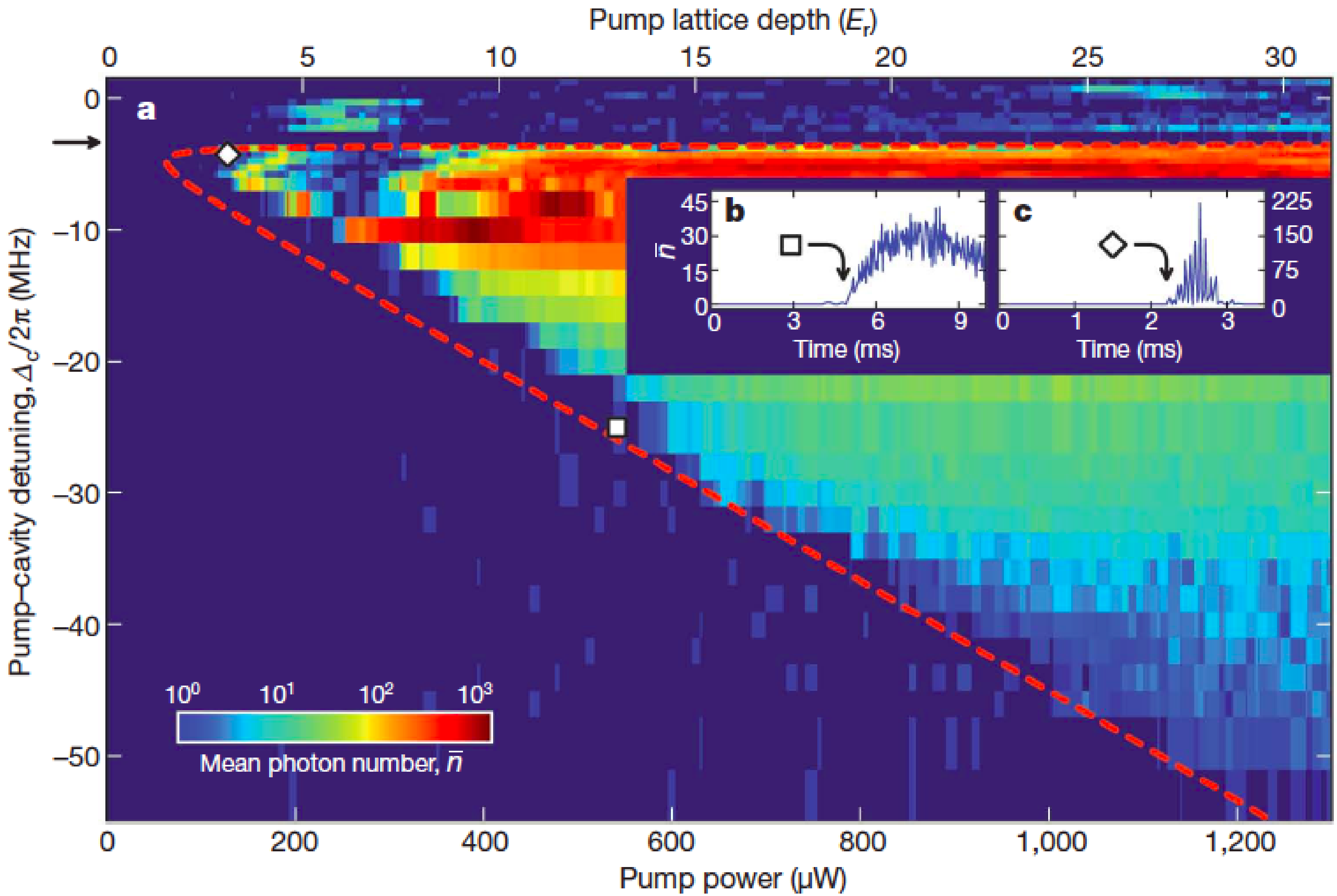
Red far-detuned pumping lasers

$$\Delta_a = \omega_p - \omega_a$$

Cavity detuning $\Delta_c = \omega_p - \omega_c < 0$

K. Baumann & *el at*, Nature 464, 1301 (2010); PRL 107, 140402 (2011);
R. Mottl & *el at*, Science 336, 1570 (2012)

Superradiance in BEC



Explained by linear stability analysis of the Gross-Pitaevskii eqaution, K. Baumann *& el at*, Nature 464, 1301 (2010)

Transition related to the density correlations of the atomic gases

Superradiance in Degenerate Fermi Gases

Consider spinless fermions, no direct interatomic interactions

$$H=\int d\boldsymbol{r}[\psi^{+}(\boldsymbol{r})h_0\psi(\boldsymbol{r})]-\Delta_c a^{+}a$$

$$h_0=h_{at}+\eta(\boldsymbol{r})(a^{+}+a)+U(\boldsymbol{r})a^{+}a,$$

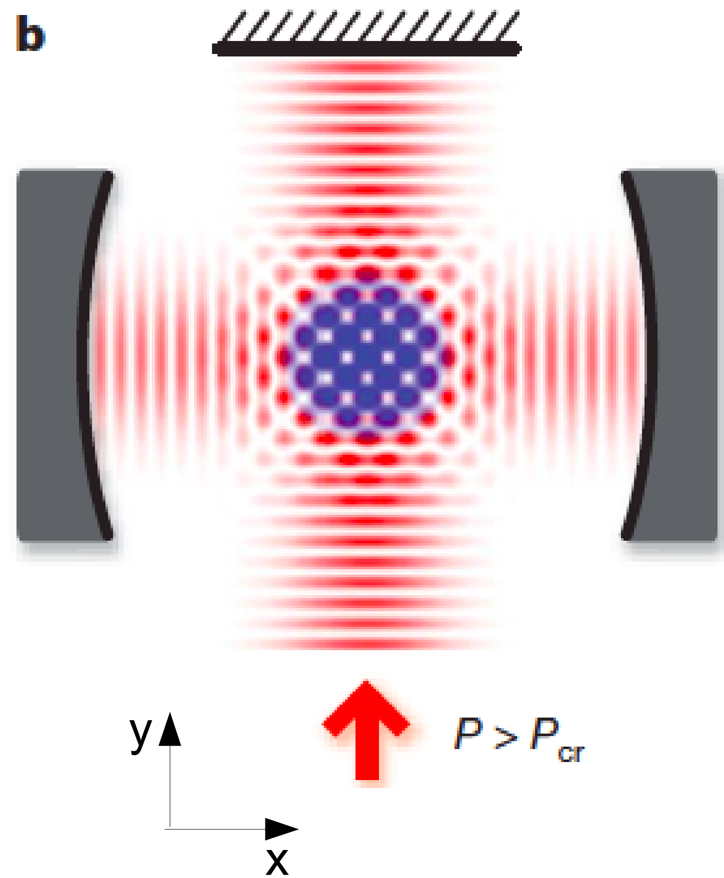
$$h_{at}=\frac{\boldsymbol{P}^2}{2\mathrm{m}}+\frac{\Omega_p^2}{\Delta_a}\cos^2(k_0 y)$$

$$\eta(\boldsymbol{r})=\eta_0\cos(k_0 x)\cos(k_0 y),$$

$$U(\boldsymbol{r})=\frac{g^2}{\Delta_a}\cos^2(k_0 x)$$

Cavity mode

Pumping laser mode



$$\eta_0=\frac{g\,\Omega_p}{\Delta_a}$$

Pumping laser
Rabi frequency

Cavity mode coupling

Nature of Superradiance

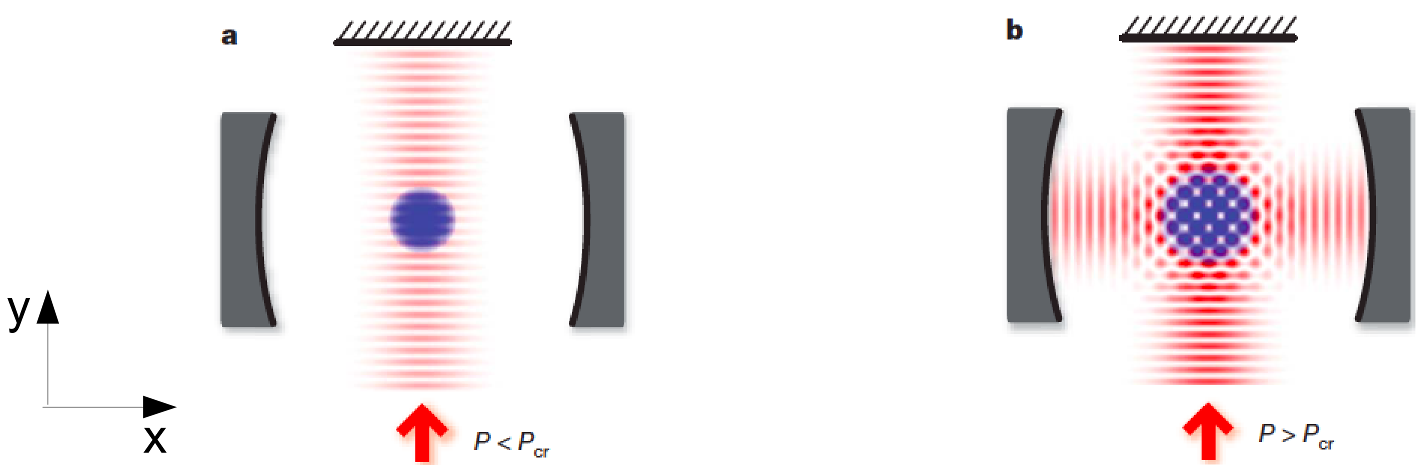
$$H=\int d\boldsymbol{r}[\psi^+(\boldsymbol{r})h_0\psi(\boldsymbol{r})]-\Delta_c a^+ a$$

$$h_0=h_{at}+\underline{\eta(\boldsymbol{r})(a^+ + a)+U(\boldsymbol{r})a^+ a}\,,\quad \eta(\boldsymbol{r})=\eta_0\cos(k_0 x)\cos(k_0 y)$$

Equation of motion: $i\dfrac{\partial a}{\partial t}=-(\tilde{\Delta}_c+i\kappa)a+\eta_0\Theta$

Density order: $\Theta=\int d\boldsymbol{r}\, n(\boldsymbol{r})\eta(\boldsymbol{r})/\eta_0$

Effective cavity detuning $\tilde{\Delta}_c=\Delta_c-\int d\boldsymbol{r}\, n(\boldsymbol{r})U(\boldsymbol{r})<0$



Mean Field Theory

Order parameter: $\Theta = \int d\boldsymbol{r} \langle n(\boldsymbol{r}) \rangle \cos(k_0 x) \cos(k_0 y)$

Steady solution: $0 = i \dfrac{\partial \langle a \rangle}{\partial t} = -(\tilde{\Delta}_c + i\kappa)\langle a \rangle + \eta_0 \Theta$

$$\langle a \rangle = \frac{\eta_0 \Theta}{\tilde{\Delta}_c + i\kappa}$$

Free energy:

$$F = -\frac{1}{\beta} \ln Tr\, e^{-\beta H} = -\left[\frac{\tilde{\Delta}_c}{\tilde{\Delta}_c^2 + \kappa^2} + \eta_0^2 \chi \frac{4 \tilde{\Delta}_c^2}{(\tilde{\Delta}_c^2 + \kappa^2)^2} \right] (\eta_0 \Theta)^2$$

$$\chi = -\frac{1}{2\beta \eta_0^2} Tr[G_0 \eta(\boldsymbol{r}) G_0 \eta(\boldsymbol{r}')] > 0$$

Single particle Green's function

Density susceptibility to modulation $\eta(\boldsymbol{r})/\eta_0$

Transition Condition

$$\eta_0^{cr} = \frac{1}{2}\sqrt{\frac{\tilde{\Delta}_c^2 + \kappa^2}{(-\tilde{\Delta}_c)\chi}} \qquad \eta_0 = \frac{g\,\Omega_p}{\Delta_a}$$

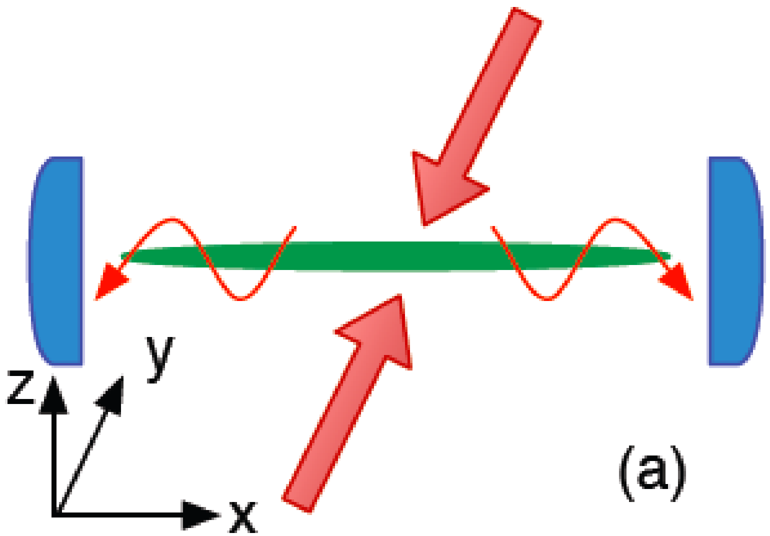
In terms of the single particle states $\phi_{\boldsymbol{k}}$

Single particle distribution function

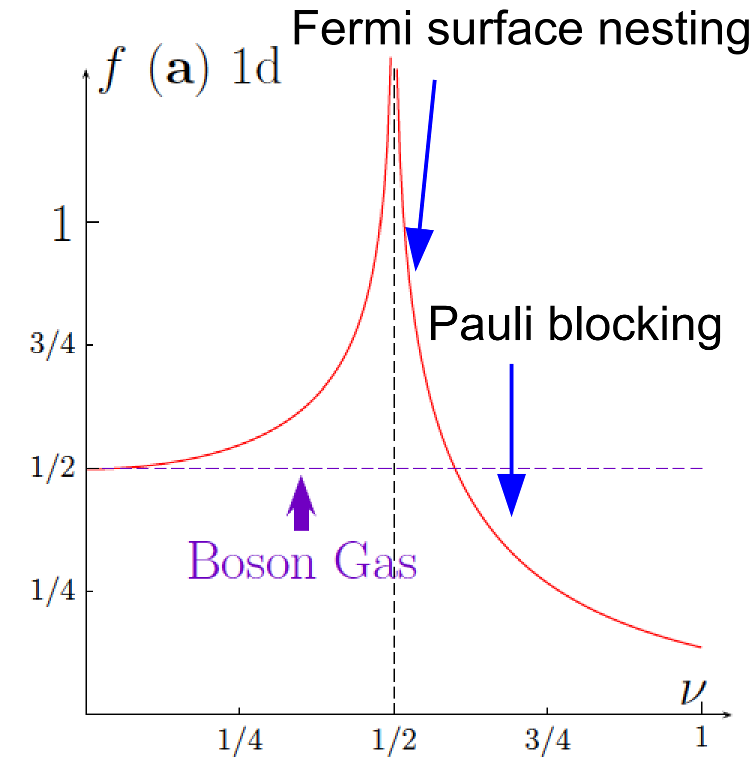
$$\chi = \frac{1}{2\eta_0^2}\sum_{\boldsymbol{k},\boldsymbol{k}'}\left|\int d\boldsymbol{r}\,\phi_{\boldsymbol{k}}^*(\boldsymbol{r})\,\phi_{\boldsymbol{k}'}(\boldsymbol{r})\,\eta(\boldsymbol{r})\right|^2 \frac{n(\epsilon_{\boldsymbol{k}}) - n(\epsilon_{\boldsymbol{k}'})}{\epsilon_{\boldsymbol{k}} - \epsilon_{\boldsymbol{k}'}}$$

Also applies to BEC

1d @ T=0



$$\eta(\boldsymbol{r}) \sim \cos(k_0 x)$$

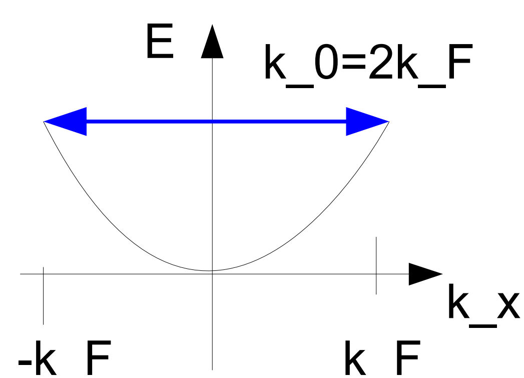


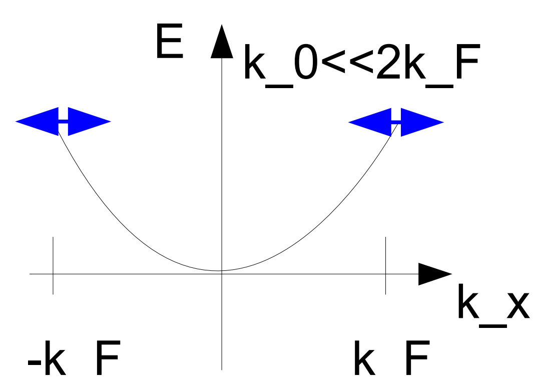
Normalized and dimensionless susceptibility

Filling =n/2k_0

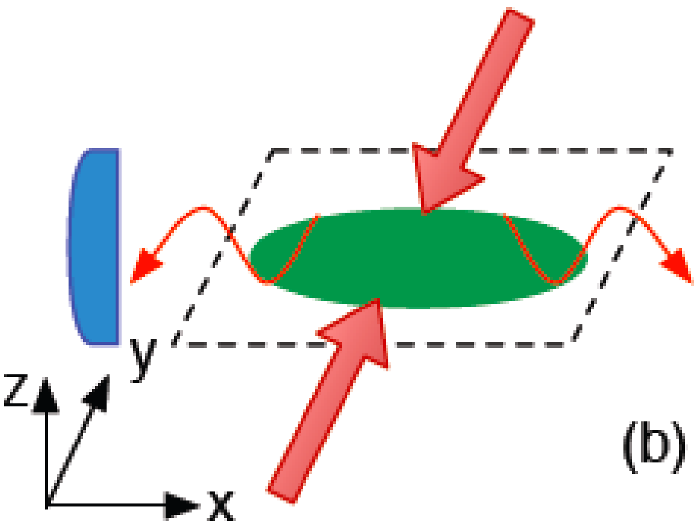
$$f = \chi E_r / N_{at} \qquad E_r = k_0^2 / 2\mathrm{m}$$

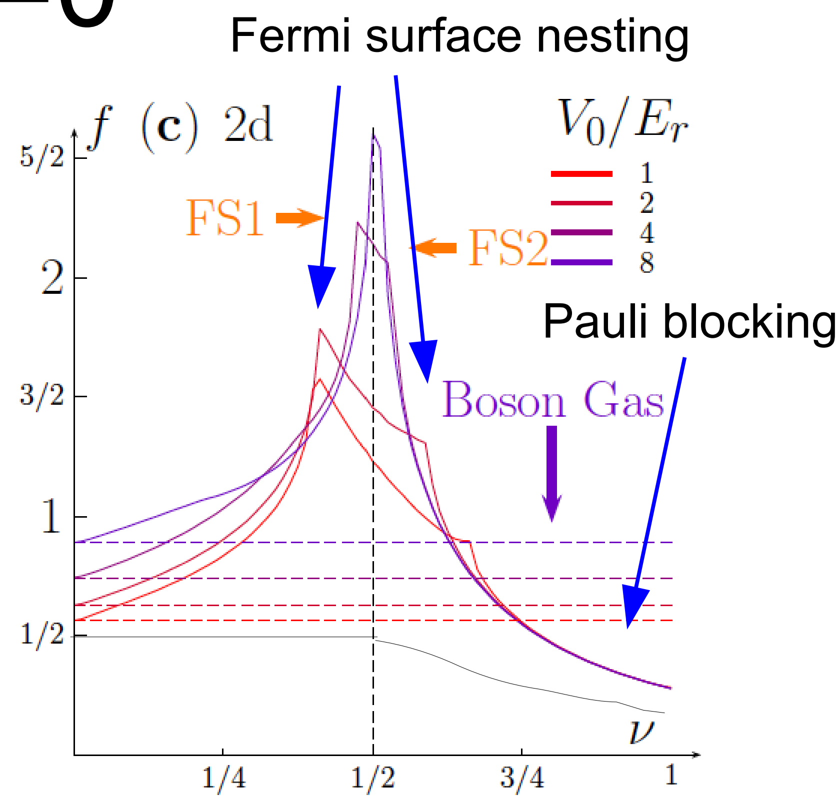
$$f = \frac{k_0}{8\mathrm{k}_F} \ln \left| \frac{k_0 + 2\mathrm{k}_F}{k_0 - 2k_F} \right|$$





2d @ T=0

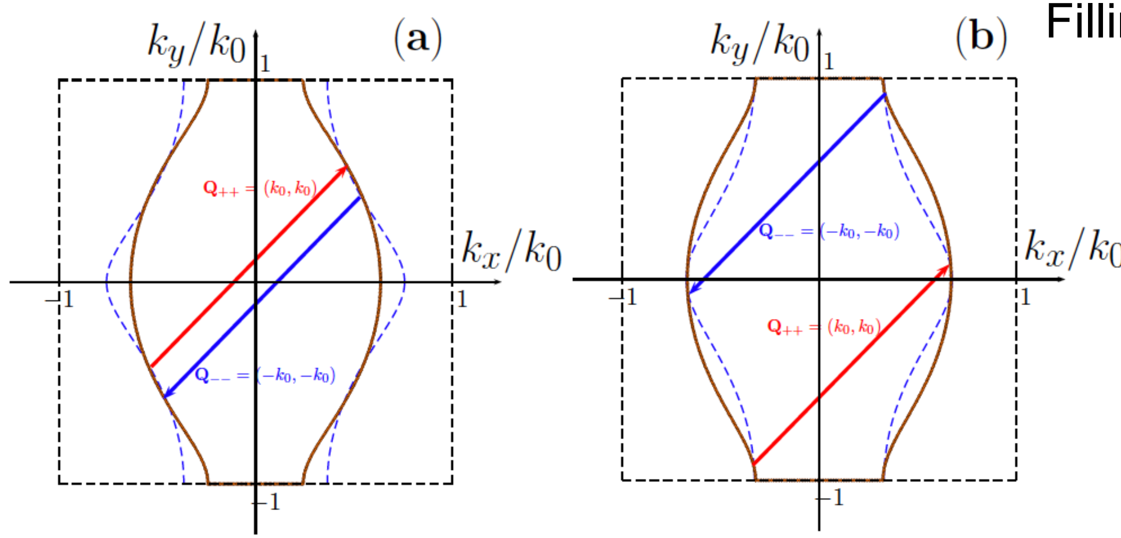




$$\eta(\boldsymbol{r}) \sim \cos(k_0 x)\cos(k_0 y)$$

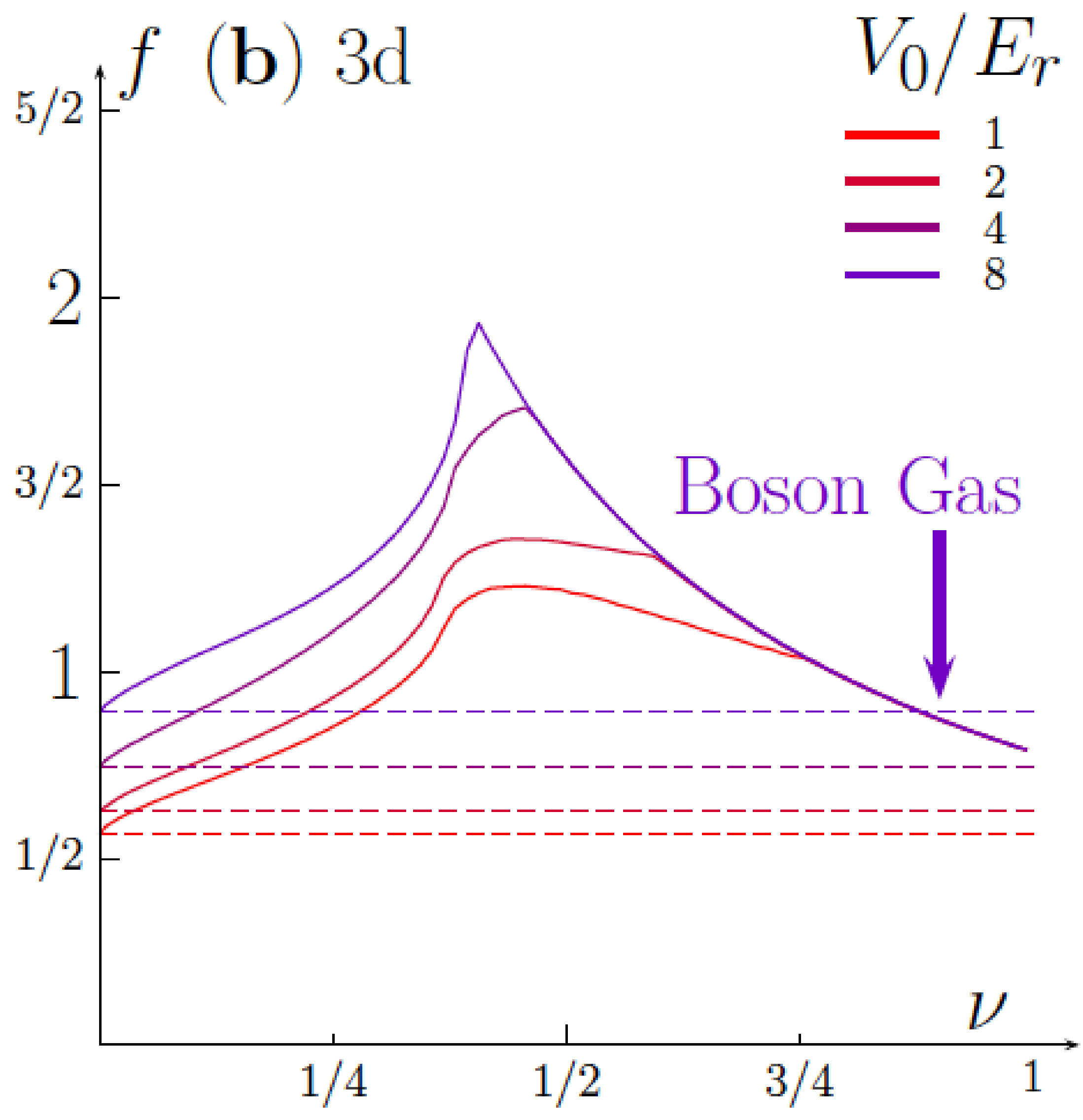
$$h_{at} = \frac{\boldsymbol{P}^2}{2\mathrm{m}} + \frac{\Omega_p^2}{\Delta_a}\cos^2(k_0 y)$$

$$V_0 = \frac{\Omega_p^2}{\Delta_a}$$

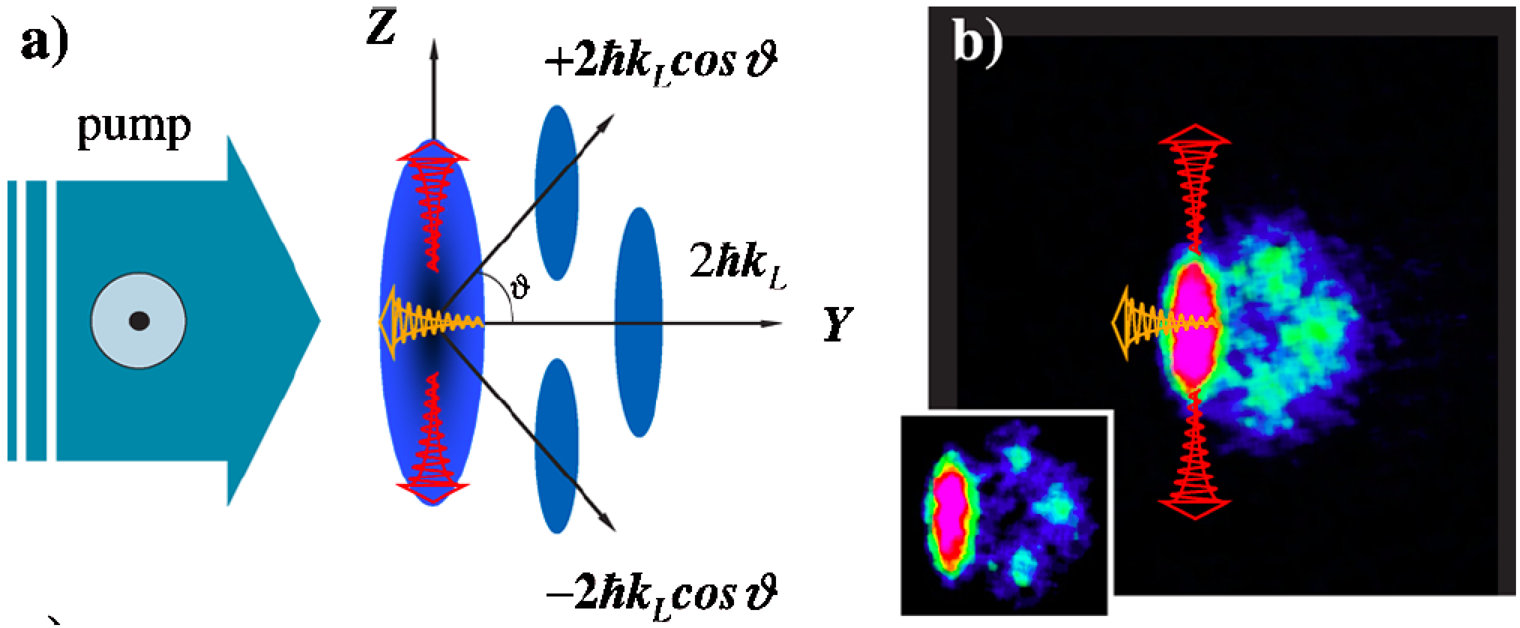


Filling =n/4k_0^2

3d @ T=0



Superradiance in free space



Bosons: Ketterle's group
Science 285, 571 (1999)

Fermions: Zhang Jing's group
PRL 106, 210401 (2011)

Phase Diagram for 3d

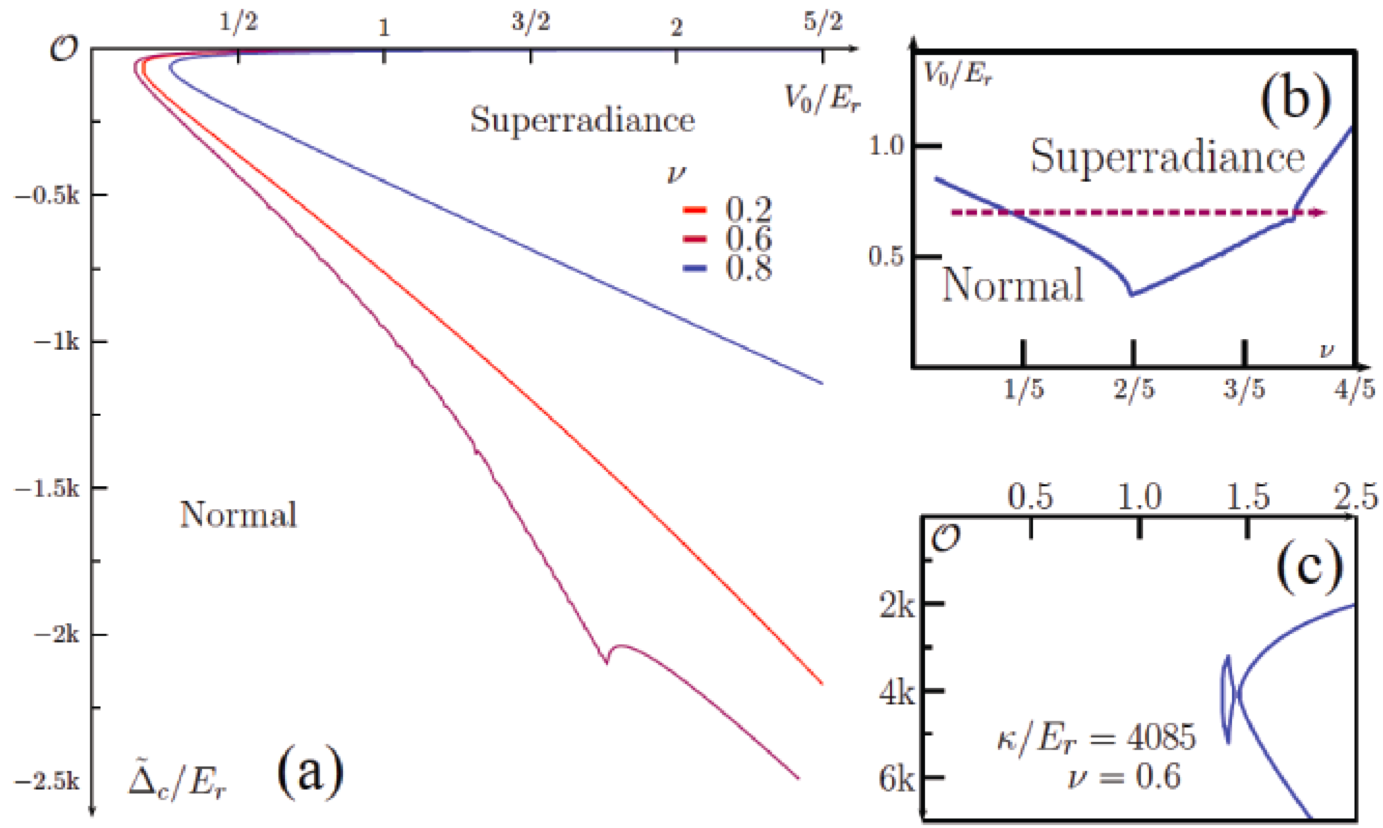
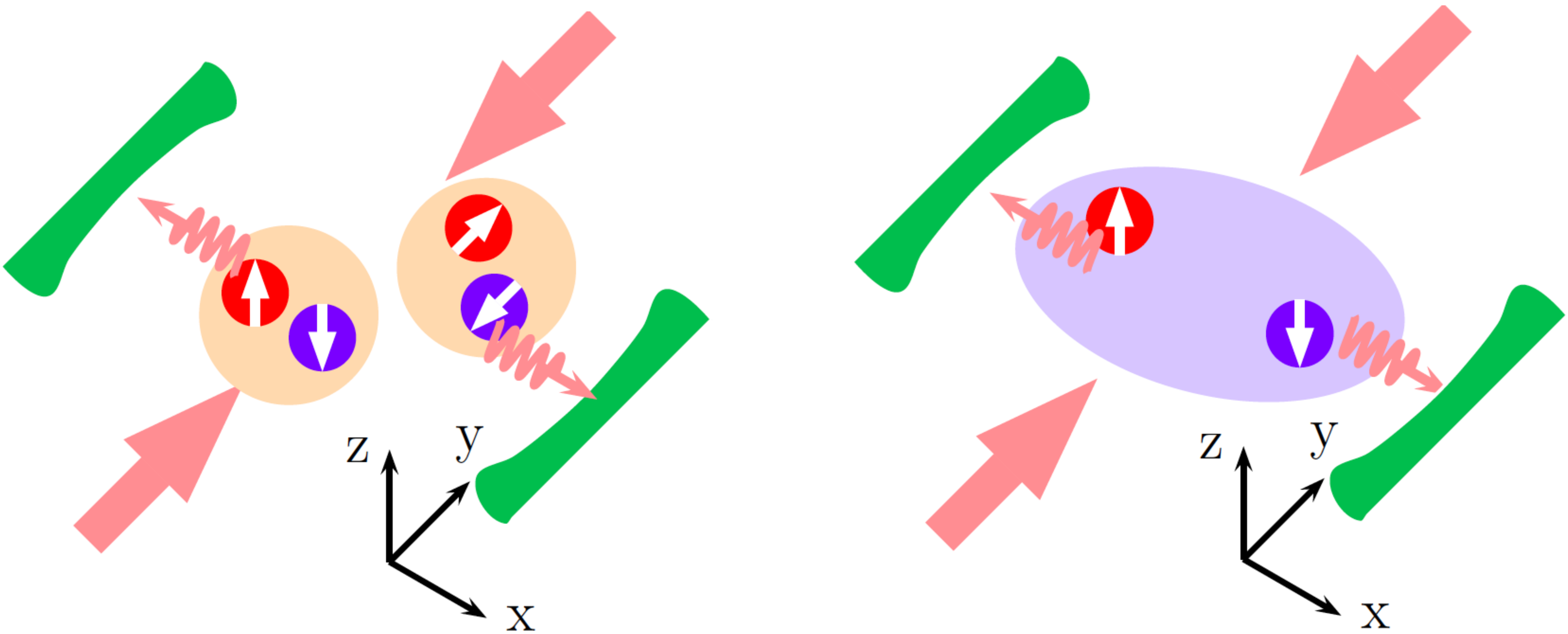


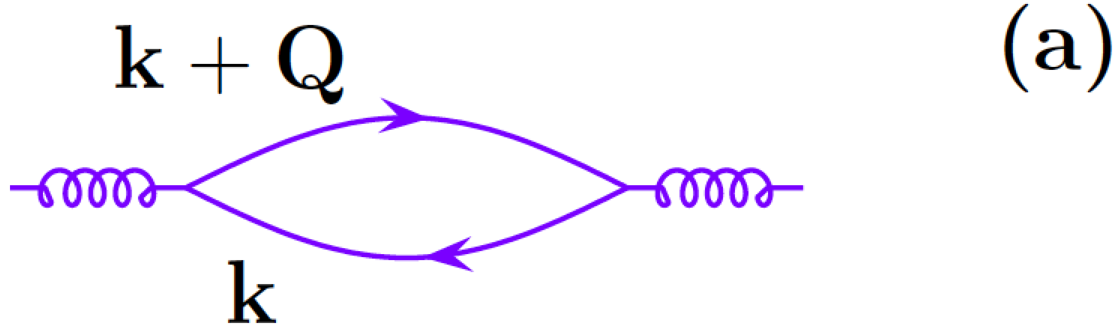
FIG. 4: (a) and (c): The phase diagram for two-dimension case, in terms of effective detuning $\tilde{\Delta}_c/E_r$ and pumping lattice depth V_0/E_r. Different lines in (a) represent phase boundary with different fillings. (b) Critical V_0/E_r as a function of filling ν for $\tilde{\Delta}/E_r$ fixed at 2×10^3. $\kappa/E_r = 250$ for (a) and (b); $\kappa/E_r = 4085$ for (c) and $U_0 N_{at}/E_r = 1 \times 10^3$ for (a-c).

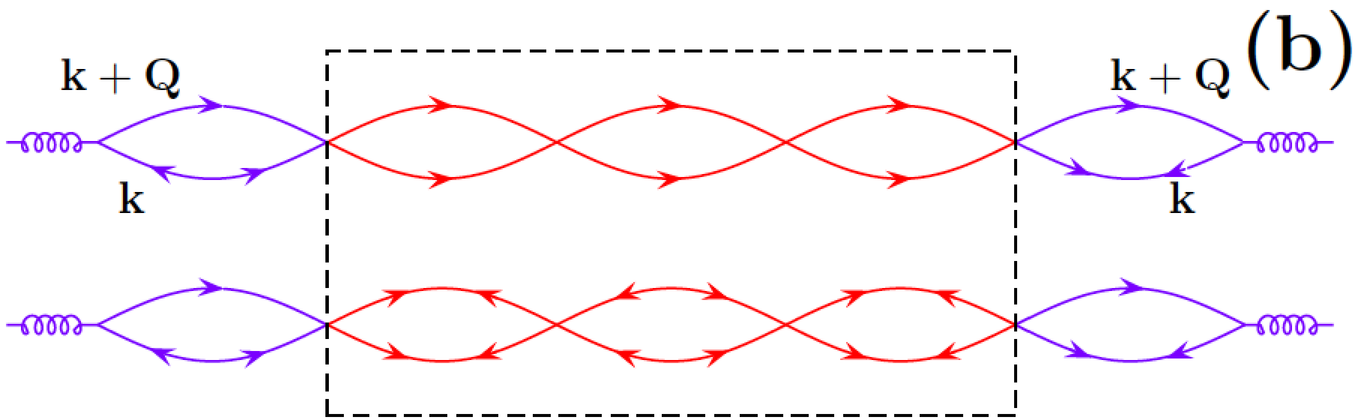
Ongoing Project



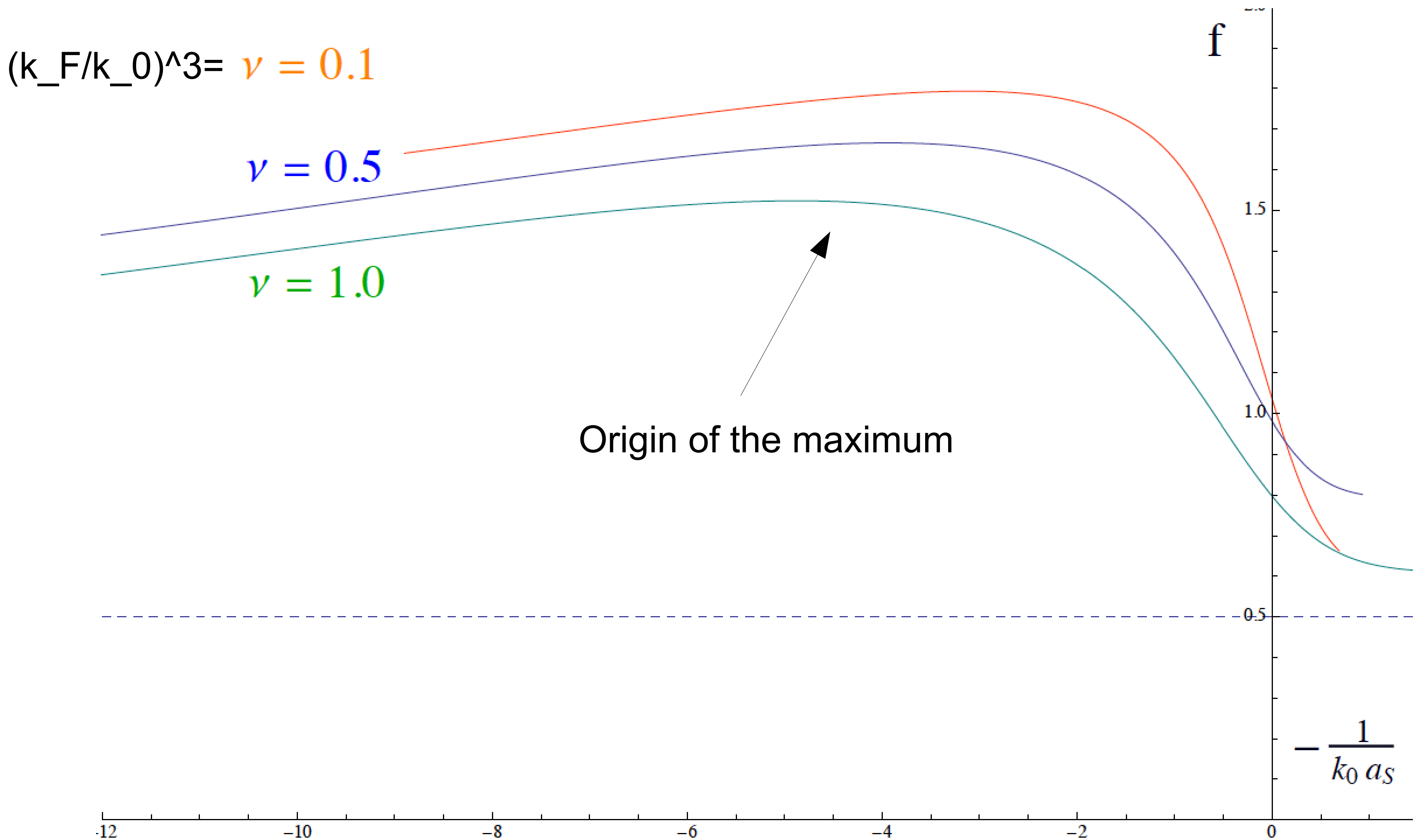
Interplay between BEC-BCS crossover and superradiance

$$F = -\frac{1}{\beta}\ln Tr\, e^{-\beta H} = -\left[\frac{\tilde{\Delta}_c}{\tilde{\Delta}_c^2+\kappa^2} + \eta_0^2 \chi \frac{4\tilde{\Delta}_c^2}{(\tilde{\Delta}_c^2+\kappa^2)^2}\right](\eta_0 \Theta)^2$$





Preliminary Result



Thank you