Universal quantum transport in ultracold Fermi gases

How slowly can spins diffuse?

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Transport in quantum fluids

Can mass flow without friction?

Shear viscosity to entropy ratio: expt minimum values \( \eta/s \sim 0.4 \ldots 0.8 \hbar/k_B \)

Kinetic theory: \( \frac{\eta}{s} \sim \frac{\ell_{\text{mfp}}}{\ell} \frac{\hbar}{k_B} \gtrsim \mathcal{O}(1) \frac{\hbar}{k_B} \) extrapolated from weak coupling

gauge-gravity duality: \( \frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \) perfect fluidity Kovtun, Son & Starinets 2005 conjectured as universal lower bound

Transport near quantum critical point (QCP): incoherent relaxation
Strongly interacting Fermi gas

- **dilute** gas of ↑ and ↓ fermions, |r_0| ≪ ℓ contact interaction: **universal**

\[ H = \int d\mathbf{x} \sum_{\sigma = \uparrow, \downarrow} \psi^\dagger_\sigma \left( -\frac{\hbar^2 \nabla^2}{2m} - \mu_\sigma \right) \psi_\sigma + g_0 \psi^\dagger_\uparrow \psi^\dagger_\downarrow \psi_\downarrow \psi_\uparrow \]

- **strong** s-wave scattering, |a| ≫ ℓ (Feshbach resonance); scale invariance

**superfluid of fermion pairs below T_c/T_F = 0.167(13)**
Ku et al. Science 2012

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**Figure 1**
Phase diagram of the BCS to BEC crossover as a function of the dimensionless attraction. The global phase diagram is based on ref. 1/0. The experiment reported by Gaebler et al. Science 2012.

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Ku et al. Science 2012
Quantum critical point

- resonant fixed point is **Quantum Critical Point** (QCP) at \( T=0, \mu=0, 1/a=0 \) 
  Nikolic & Sachdev 2007

- abrupt change of ground state at QCP
  density \( n \) is **order parameter**: vacuum for \( T=0, \mu<0 \)
  gapless excitations above QCP: affect measurements in quantum critical regime

![Diagram showing the phase diagram of a quantum critical point with temperature \( T \) and chemical potential \( \mu \). The phase diagram includes regions labeled dilute classical gas, vacuum, quantum critical regime, and superfluid, with a critical point \( T_C \approx 0.4\mu \) at unitarity \( 1/a=0 \).]
Universal properties

- at unitarity $1/a=0$ **scale invariance**: properties depend only on $\mu/T$ (“angle”) 
  [Zhang+ Science 2012]

- e.g. equation of state $n = \lambda_T^{-3} f_n(\mu/T)$

  measured by Zwierlein group (2012), computed using Bold Diagrammatic MC

- **quantum critical regime** above QCP:
  \[
  \lambda_T \approx n^{-1/3} \quad (T_c \lesssim T \lesssim T_F)
  \]

  quantum and thermal fluctuations equally important, interplay challenging

  temperature only available scale for **incoherent relaxation**: [Sachdev 1999]
  \[
  \frac{\hbar}{\tau_\eta} = \mathcal{O}(1) k_B T \quad \Rightarrow \quad \frac{\eta}{s} = \frac{2}{5} T \tau_\eta \approx 0.7 \frac{\hbar}{k_B} \quad (\mu = 0)
  \]

  [large-N exp’n Enss 2012]
Luttinger-Ward approach

- repeated particle-particle scattering dominant in dilute gas:

\[ \text{self-consistent T-matrix} \quad \text{Haussmann 1993, 1994; Haussmann et al. 2007} \]

self-consistent fermion propagator
(300 momenta / 300 Matsubara frequencies)

- spectral function \( A(k,\epsilon) \) at \( T_c \)

\[ \text{works above and below } T_c; \text{ directly in continuum limit} \]

\[ T_c=0.16(1) \text{ and } \xi=0.36(1) \]

agree with experiment

conserving: exactly fulfills scale invariance and Tan relations

\[ \text{Enss PRA 2012} \]
Transport in linear response

- shear viscosity from stress correlations (cf. hydrodynamics),
  \[ \eta(\omega) = \frac{1}{\omega} \text{Re} \int_0^\infty dt \, e^{i\omega t} \int d^3x \left\langle \left[ \hat{\Pi}_{xy}(\mathbf{x}, t), \hat{\Pi}_{xy}(0, 0) \right] \right\rangle \]
  with stress tensor \( \hat{\Pi}_{xy} = \sum_{\mathbf{p}, \sigma} \frac{p_x p_y}{m} c_{\mathbf{p}\sigma}^{\dagger} c_{\mathbf{p}\sigma} \) (cf. Newton \( \frac{\partial v_x}{\partial y} \))

- correlation function (Kubo formula): Enss, Haussmann & Zwerger, Annals Physics 2011
  \[ \eta(\omega) = \begin{array}{c}
  \text{(S)} \\
  \text{(MT)} \\
  \text{(AL)}
  \end{array} \]
  (resummed to infinite order)

- transport via fermions and bosonic molecules: very efficient description, satisfies conservation laws, scale invariance and Tan relations Enss PRA 2012

- assumes no quasiparticles: beyond Boltzmann kinetic theory, works near Tc; includes pseudogap and vertex corrections
High-energy tails in stress correlation (shear viscosity)

exact viscosity sum rule (nonperturbative check):

\[ \frac{2}{\pi} \int_{0}^{\infty} d\omega \left[ \eta(\omega) - \text{tail} \right] = P - \frac{C}{4\pi ma} \]

Taylor & Randeria 2010; Enss, Haussmann & Zwerger 2011; Enss 2013
Shear viscosity/entropy of the unitary Fermi gas

\[ \frac{\eta}{s} \sim T^{-8} \]

Luttinger-Ward theory
kinetic theory

lowest friction of any nonrelativistic quantum fluid

agrees with large-N transport calculation
Enss PRA 2012

Shear viscosity/entropy of the unitary Fermi gas

Enss, Haussmann & Zwerger 2011

cf. theory: Bruun, Massignan, Schäfer, Smith, ...

experiment: Cao+ Science 2011
Spin transport with ultracold gases

- **experiment**: spin-polarized clouds in harmonic trap
- **strongly interacting gas** [movie courtesy Martin Zwierlein]:

Spin diffusion

- scattering conserves total \( \uparrow + \downarrow \) momentum: mass current preserved
- but changes relative \( \uparrow - \downarrow \) momentum: spin current decays

![Spin diffusion graph](image_url)

Sommer et al. 2011

Spin diffusion
Spin conductivity and spin susceptibility

use Einstein relation

\[ D_s = \frac{\sigma_s}{\chi_s} \]

Spin conductivity:

\[ \sigma_s \text{ m}E_F/\hbar n \]

spin susceptibility:

\[ \chi_n, \chi_s \text{ Sommer et al. (2011)} \]

\[ \chi_n, \chi_s \text{ Luttinger-Ward} \]

\[ \chi_n, \chi_s \text{ free Fermi gas} \]

medium effects important:

- large-N transport calculation Enss PRA 2012
- in two dimensions Enss, Küppersbusch, Fritz PRA 2012

related work on spin transp.:

Bruun 2011; Duine et al. 2011; Mink et al. 2012, 2013
Spin diffusivity

- obtain diffusivity from Einstein relation, \( D_s = \frac{\sigma_s}{\chi_s} \)

![Graph showing spin diffusivity vs temperature]

(experiment rescaled from trap to infinite homogeneous box)

- Quantum Monte Carlo simulation for finite range interaction: \( D_s \gtrsim 0.8 \frac{\hbar}{m} \)

Wlazlowski et al. PRL 2013

Enss & Haussmann PRL 2012
Longitudinal vs **transverse** spin diffusion

**spin polarized**

Mullin & Jeon 1992
Spin-echo experiment (Thywissen group, Toronto)

\[ M_\perp(t) = -i \exp[-t^3/24\tau^3] \]

Bardon+ Science 334, 722 (2014)

2D: Koschorreck et al. Nature Physics 2013
Spin diffusion in kinetic theory

- Local magnetization vector and gradient:
  \[ \mathcal{M}(r, t) = \mathcal{M}(r, t) \hat{e}(r, t) \]
  \[ \frac{\partial \mathcal{M}}{\partial r_i} = \frac{\partial \mathcal{M}}{\partial r_i} \hat{e} + \mathcal{M} \frac{\partial \hat{e}}{\partial r_i} \]

- Boltzmann equation for spin distribution function:
  \[ \frac{D \sigma_p}{Dt} \equiv \frac{\partial \sigma_p}{\partial t} - \sum_i v_{pi} \frac{\partial \mathcal{M}}{\partial r_i} \hat{e} \sum_\sigma t_\sigma \frac{\partial n_{p\sigma}}{\partial \epsilon_p} \]
  \[ + \sum_i v_{pi} \frac{\partial \hat{e}}{\partial r_i} (n_{p+} - n_{p-}) + \Omega \times \sigma_p = \left( \frac{\partial \sigma_p}{\partial t} \right)_{\text{coll}} \]

- Many-body T-matrix in collision integral and spin rotation
  Enss 2013
  Derived as leading order in large-N expansion

Spin-rotation effect


\[ M^+(\mathbf{r}, t) = M_x + i M_y : \quad \frac{\partial M^+}{\partial t} \simeq -i \Omega_0(\mathbf{r}) M^+ + D_\perp (1 + i \mu M_z) \nabla^2 M^+ \]

Leggett-Rice spin-rotation effect: complex diffusion constant, spin waves; spin current precesses around effective molecular field:

\[ \mu = -\Omega_{\text{mf}} \tau_\perp \]

Enss PRA 2013
In the weak-coupling limit, the scattering cross section is given by the fusivity in the Boltzmann limit [18, 19].

In two dimensions, we find

\[ I_{\text{trans}} = 0 \]

For longitudinal spin diffusion, we have

\[ I_{\text{long}} = \frac{\alpha}{\tau} \]

The angular average yields

\[ I_{\text{long}} = \frac{I_{\text{trans}}}{2} \]

The longitudinal scattering rate in the unpolarized case derivation one obtains the longitudinal scattering collision integral (16) is diagonal, and following the standard derivation one obtains the longitudinal scattering rate in the unpolarized case. 

The scattering time and diffusion coefficient are given by

\[ \tau = \frac{1}{\lambda q} \]

\[ D = \frac{\lambda^2}{2} \]

If the diffusion time is much smaller than the scattering time, the scattering time converges toward the transverse scattering time.

In Fig. 1, the transverse and longitudinal spin diffusivities have been computed with the vacuum integral is computed using the vacuum medium.

In the unitary Fermi gas in three dimensions, the collision integral is computed using the vacuum medium.

In the normal Fermi liquid within Born approximation, the transverse and longitudinal diffusivities have been computed with the vacuum medium.

As explained in section II A, in a systematic expansion to leading order, one has to use the quantum degenerate regime where the transverse diffusivity decreases at low temperatures and reaches a finite value, while the longitudinal diffusivity which due to Pauli blocking diverges as the temperature approaches zero. This is in marked contrast to the longitudinal diffusivity for a normal Fermi liquid within Born approximation, where the diffusivity much smaller!}

In Fig. 1, the diffusivities have been computed with the vacuum medium.

Bardon+ Science 334, 722 (2014)
Transverse spin diffusivity (2D)

![Graph showing transverse spin diffusivity (2D) as a function of interaction strength and temperature.](image)

**Vacuum scattering**

Enss PRA 2013

**Dependence on interaction and importance of medium effects**

cf. $\eta$: Enss, Küppersbusch & Fritz PRA 2012

trap: Chiacchiera, Davesne, Enss & Urban, PRA 2013
Conclusion and outlook

- **lowest friction at strong interaction:** almost perfect fluidity near QCP
  large-N: Enss, PRA 86, 013616 (2012)

- **quantitative understanding of spin diffusion:**
  Luttinger-Ward transport calculation (tail, near Tc)
  slowest **longitudinal** spin diffusivity $D_s \gtrsim 1.3 \hbar/m$
  Enss & Haussmann, PRL 109, 195303 (2012)

- **transverse spin diffusion:**
  $D_\perp$ can be much lower than $D_\parallel$ in **degenerate, polarized** gas;
  Leggett-Rice **spin-rotation** effect
  Enss, PRA 88, 033630 (2013)

- **outlook:** spin-rotation effect (Thywissen group)
  2D Fermi gas: EoS and pseudogap
  Bauer, Parish & Enss, PRL 112, 135302 (2014)
BKT-BCS crossover in 2D Fermi gas

Pressure EoS (vs Turlapov data):

Phase diagram (Tc, T*):

Tan contact density (vs Köhl data):

Spectral function/pseudogap: