The Bose polaron

Challenge for quantum simulation with ultracold atoms

Richard Schmidt

- Harvard University / ITAMP -

Richard Schmidt and **Steffen Patrick Rath**

Phys. Rev. A **88**, 053632 (2013)

Cold atoms and beyond 2014
Aarhus
06/25/14

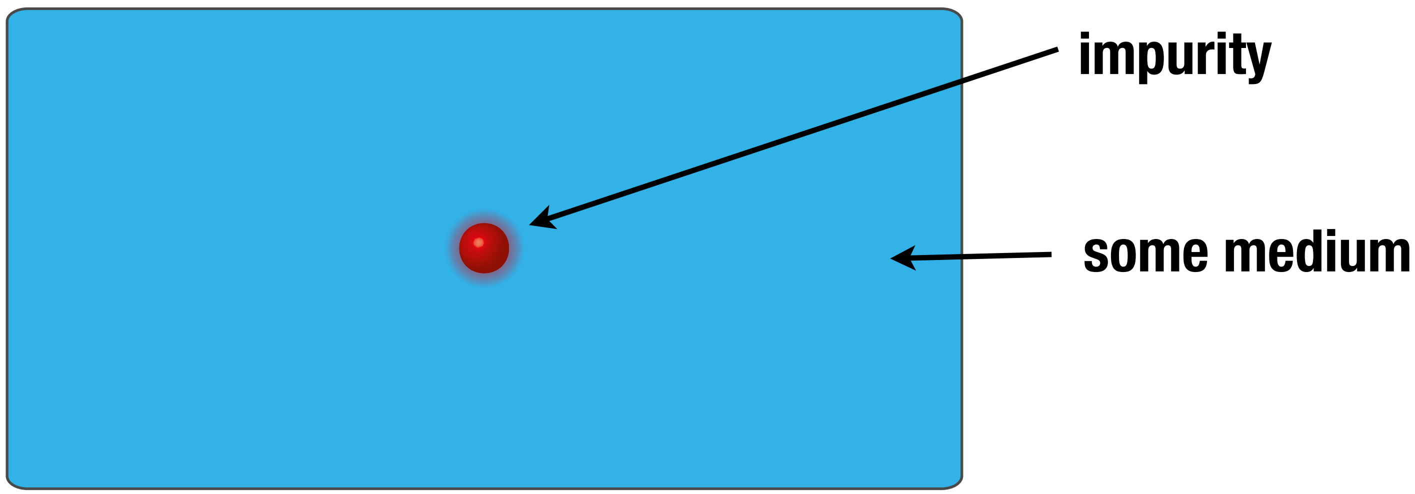
ITAMP

DFG - FOR 801

Harvard University

Impurity physics

Quantum impurities ⟷ **Paradigm of many-body physics**



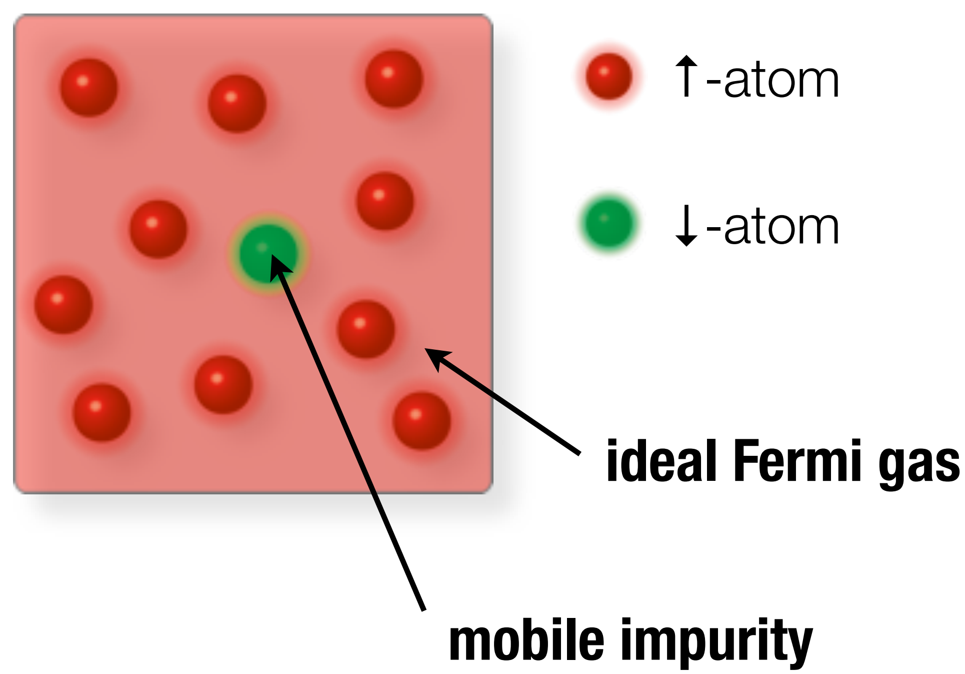
- appear in many flavors in condensed matter physics
- relatively simple system from many-body perspective: allow to advance theory in 'controlled way'
- system on the verge from few- to many-body physics

Impurity experiments with ultracold atoms

Mainly studied experimentally in cold atoms so far:

REVIEW: MASSIGNAN, ZACCANTI, BRUUN REP. PROG. PHYS. 77, 034401 (2014)

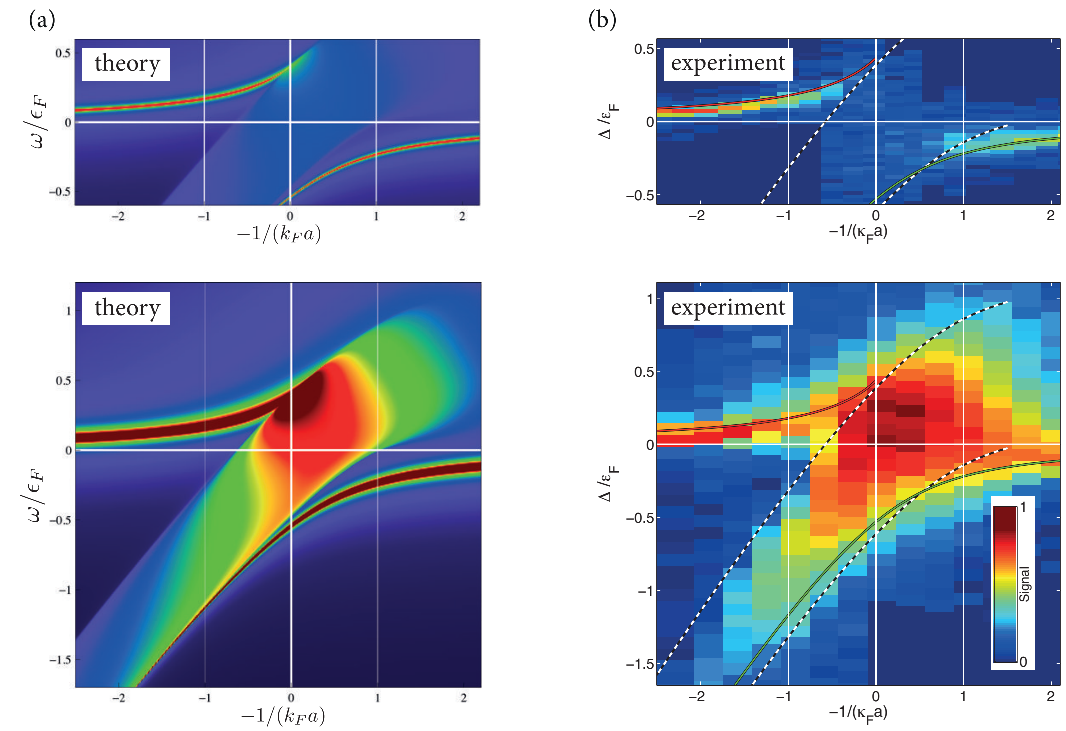
Fermi polaron *[in continuum]*



EXPERIMENTS: SCHIROTZEK ET AL., PRL 102 (2009)
KOHSTALL ET AL., NATURE 485 (2012)
KOSCHORRECK ET AL., NATURE 485 (2012)

RF response - theory vs experiment

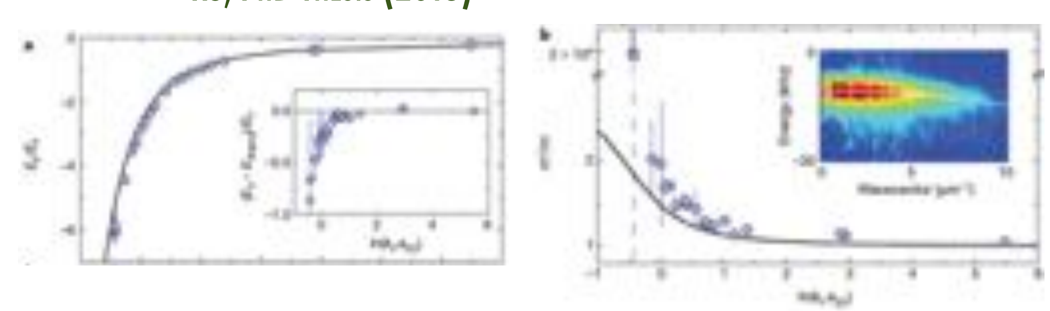
3D:



RS, ENSS, PRA 83 (2011)
MASSIGNAN, BRUUN, EPJD 65 (2011)
RS, PHD THESIS (2013)

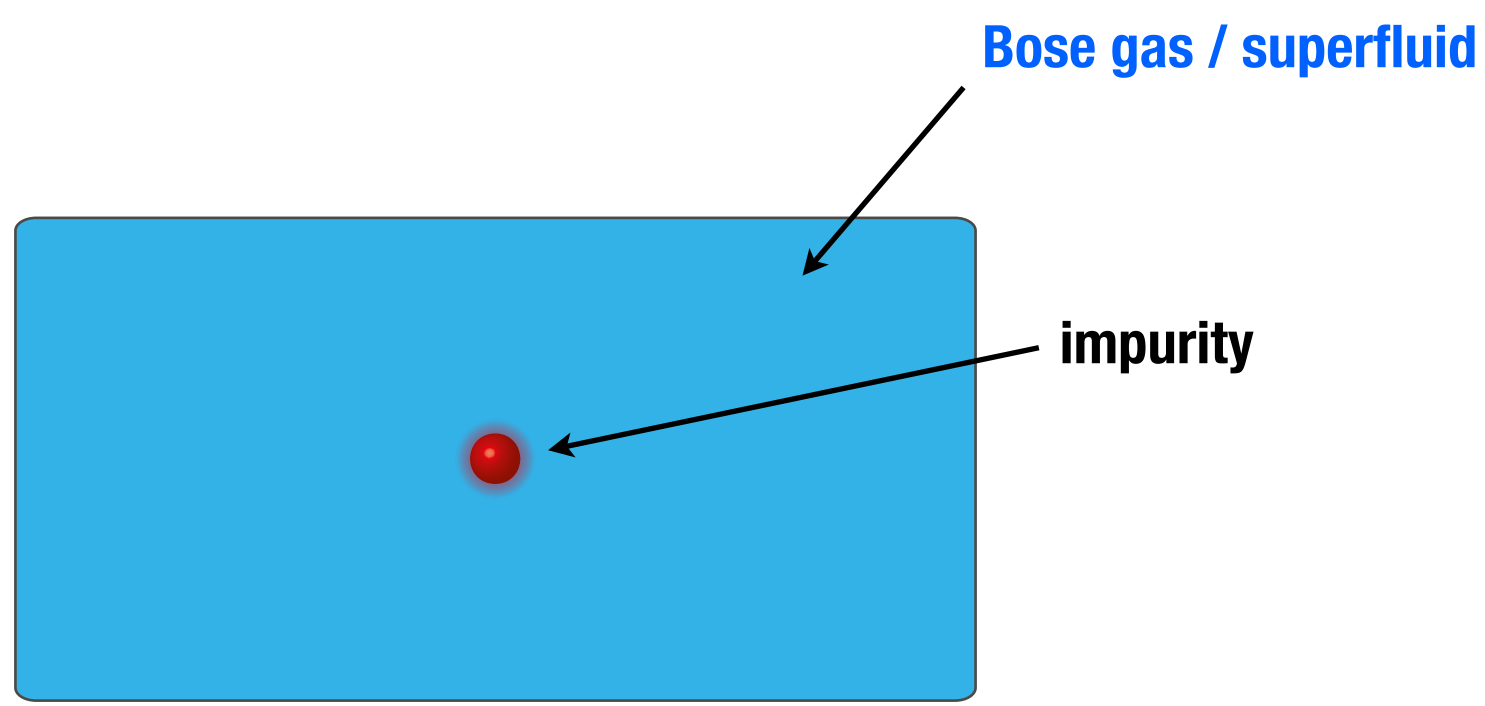
KOHSTALL ET AL., NATURE 485 (2012)

2D:



ZÖLLNER, BRUUN, PETHICK, PRA 83, 021603 (2011)
RS, ENSS, PIETILA, DEMLER, PRA 85, 021602 (2012)
KOSCHORRECK ET AL., NATURE 485 (2012)
NGAMPRUETIKORN, LEVINSEN, PARISH EPL 98 (2012)

This talk: The Bose Polaron

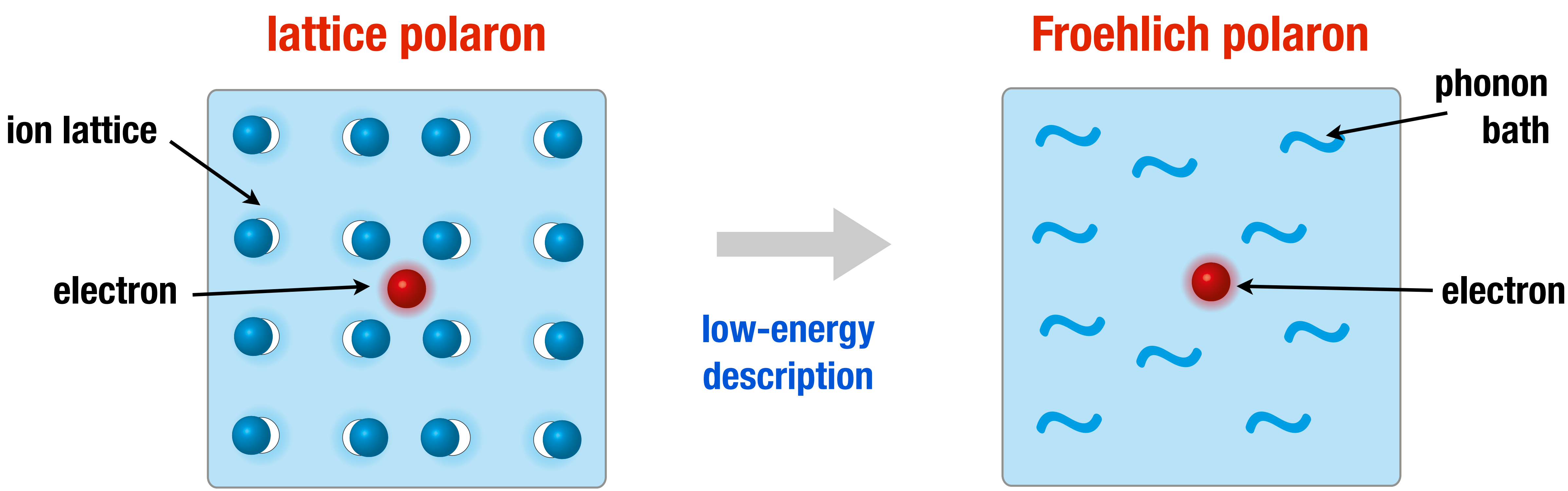


what happens if medium is Bose gas?

⇨ **Bose polaron**

A cond-mat motivation: The Froehlich polaron

a paradigm condensed matter model:



Fröhlich Hamiltonian

FRÖHLICH, ADV. PHYS. 3, 325 (1954)

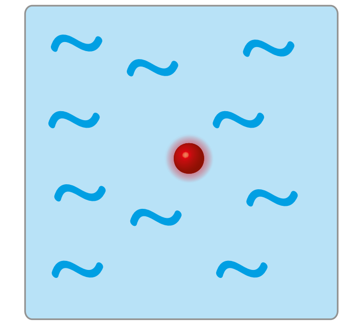
$$\hat{H} = \underbrace{\sum_{\mathbf{p}} \omega_p \hat{b}^\dagger_{\mathbf{p}} \hat{b}_{\mathbf{p}}}_{\text{phonons}} + \underbrace{\sum_{\mathbf{p}} \epsilon_{\mathbf{p}} \hat{c}^\dagger_{\mathbf{p}} \hat{c}_{\mathbf{p}}}_{\text{impurity}} + \underbrace{\sum_{\mathbf{q},\mathbf{p}} \alpha_q \, \hat{c}^\dagger_{\mathbf{p}+\mathbf{q}} \hat{c}_{\mathbf{p}} (\hat{b}^\dagger_{-\mathbf{q}} + \hat{b}_{\mathbf{q}})}_{\text{impurity-phonon interaction}}$$

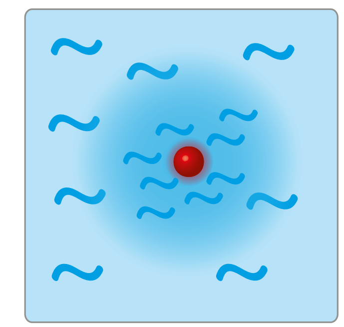
The Fröhlich polaron

Fröhlich Hamiltonian

FRÖHLICH, ADV. PHYS. 3, 325 (1954)

$$\hat{H} = \sum_{\mathbf{p}} \omega_p \hat{b}^\dagger_{\mathbf{p}} \hat{b}_{\mathbf{p}} + \sum_{\mathbf{p}} \epsilon_{\mathbf{p}} \hat{c}^\dagger_{\mathbf{p}} \hat{c}_{\mathbf{p}} + \sum_{\mathbf{q},\mathbf{p}} \underbrace{\alpha_q}_{\alpha > 0} \, \hat{c}^\dagger_{\mathbf{p}+\mathbf{q}} \hat{c}_{\mathbf{p}} (\hat{b}^\dagger_{-\mathbf{q}} + \hat{b}_{\mathbf{q}})$$





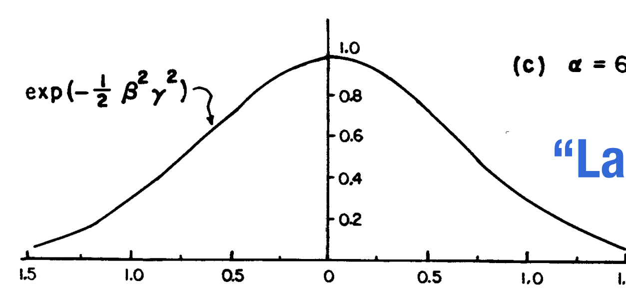
- impurity dressed by *phonon cloud* becomes the '**Fröhlich polaron**'
- enhanced effective mass, renormalized energy SEE E.G. MILLER ET AL. PHYS. REV. 127 ('62)

perturbation theory: $m^* = \dfrac{m}{1-\alpha/6} \xrightarrow[\alpha > 0]{} \infty$
large

self-localization?

- strong interactions: variational wave function

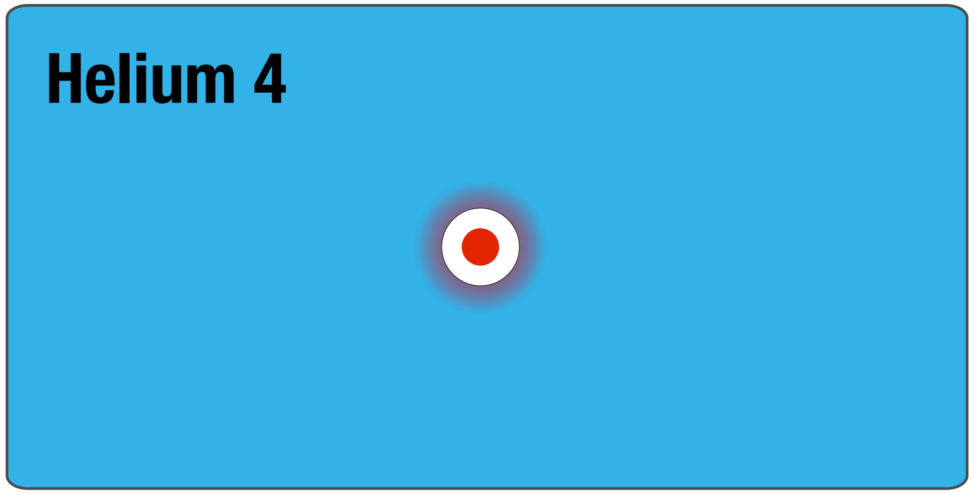
LANDAU, PEKAR, JETP 18 (1948); FEYNMAN, COHEN, PHYS. REV. 102 (1956)



"Landau Pekar polaron"

  - describes localized particle
  - yields energy smaller than pert. theory at strong coupling, further evidence of self-localization

Strong repulsion: The “bubble polaron” Kuper, Phys. Rev. 122 (1961)

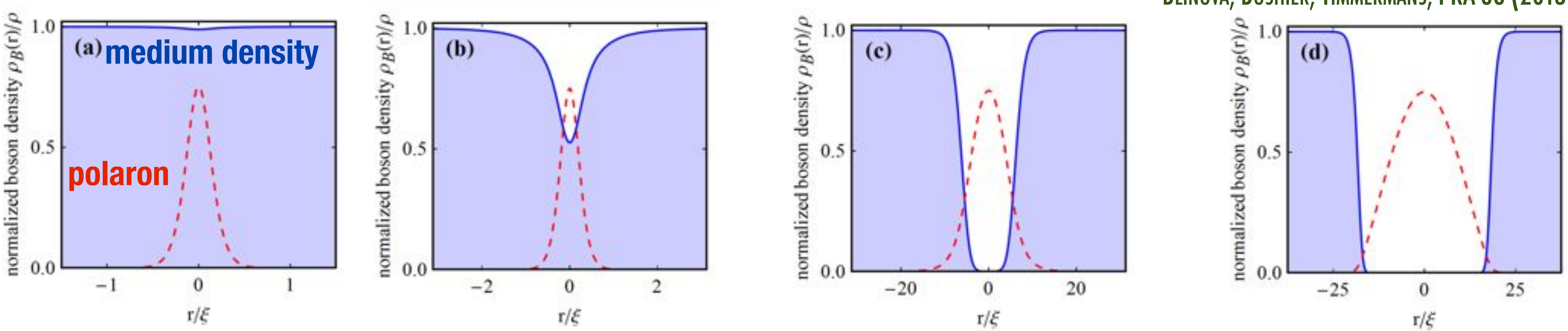


“original application”: impurity in liquid Helium

at strong repulsive interactions medium can get distorted

[not taken into account in Froehlich Hamiltonian]

Blinova, Boshier, Timmermans, PRA 88 (2013)



interaction

Landau-Pekar regime **Bubble polaron**

superfluid + strong interactions:

Realizable with ultracold atoms! → Let’s do quantum simulation!

Yet there are challenges!

The Bose polaron with ultracold atoms

impurity in Bose gas: Bose polaron

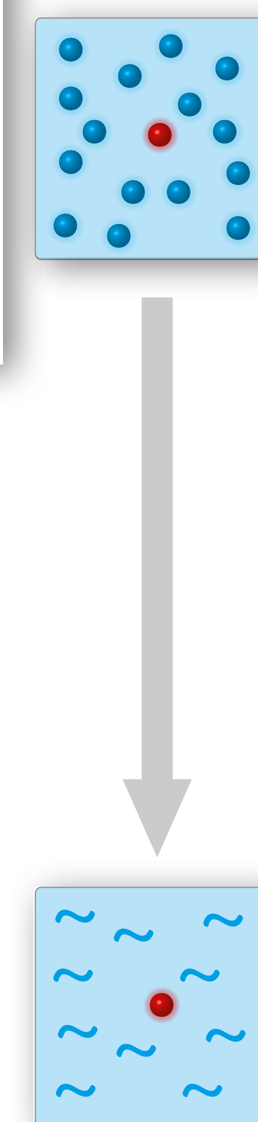
e.g. take strongly imbalanced mixture of ultracold atoms

$$S=\int d^4x\underline{\varphi^*(x)}\left(\partial_\tau-\frac{1}{2m_B}\nabla^2-\mu_B\right)\underline{\varphi(x)}\ +\frac{g_B}{2}\underline{[\varphi(x)^*\varphi(x)]^2}$$

$$+\underline{\psi^*(x)}\left(\partial_\tau-\frac{1}{2m_I}\nabla^2-\mu_I\right)\underline{\psi(x)}\ +g_{IB}\underline{\psi(x)^*\psi(x)\varphi(x)^*\varphi(x)}$$

bosons

impurity



weakly interacting BEC: Bogoliubov approximation for BEC

$$\varphi(\mathbf{x},t)=\sqrt{n_B}+\phi(\mathbf{x},t)$$

mean-field **fluctuations**

at weak coupling: **Fröhlich Hamiltonian**

$$S_{\text{eff}}=S_{\text{kin}}^{\text{Bos}}+S_{\text{kin}}^{\text{Imp}}+\underbrace{g_{IB}\int_x n_B(\mathbf{r})\,|\psi(\mathbf{r})|^2}_{\text{MF energy shift}}+g_{IB}\int\Big\{\underbrace{\sqrt{n_B}\,\psi^*_{\mathbf{k}+\mathbf{q}}\psi_{\mathbf{k}}\left(\phi_{\mathbf{q}}+\phi^*_{-\mathbf{q}}\right)}_{\text{"Fröhlich term"}}$$

Bose poloron at strong coupling

$$S_{\text{eff}} = S_{\text{kin}}^{\text{Bos}} + S_{\text{kin}}^{\text{Imp}} + g_{IB} \int \left\{ \underbrace{\sqrt{n_B}\psi_x^*\psi_x(\phi_x + \phi_x^*)}_{\text{"Fröhlich terms"}} + n_B\psi_x^*\psi_x \right.$$

$$g_{IB} = \frac{2\pi\hbar^2}{m_r} a_{IB}$$

strong *effective* phonon-impurity interaction wanted

Heiselberg et al., PRL 85 (2000)
Cucchientti, Timmermans, PRL 96 (2006)
Kalas, Blume, PRA 73 (2006)
Wang, PRL 96 (2006)
Enss, Zwerger, EPJB 68 (2009)
Tempere, Oberthaler et al., PRB 80 (2009)

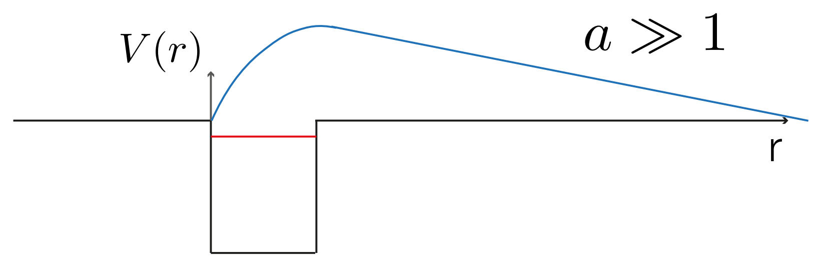
Casteels et al., PRA 83,84,86 (2011)
Casteels, Cauteren, Tempere, Devreese, Laser Phys. 21 (2011)
Casteels, Tempere, Devreese, PRA 84 (2011)
Casteels, Tempere, Devreese, PRA 86 (2012)
Dasenbrook, Komnik, PRB 87 (2013)
Blinova, Boshier, Timmermans, PRA 88 (2013)
...

Bose poloron at strong coupling

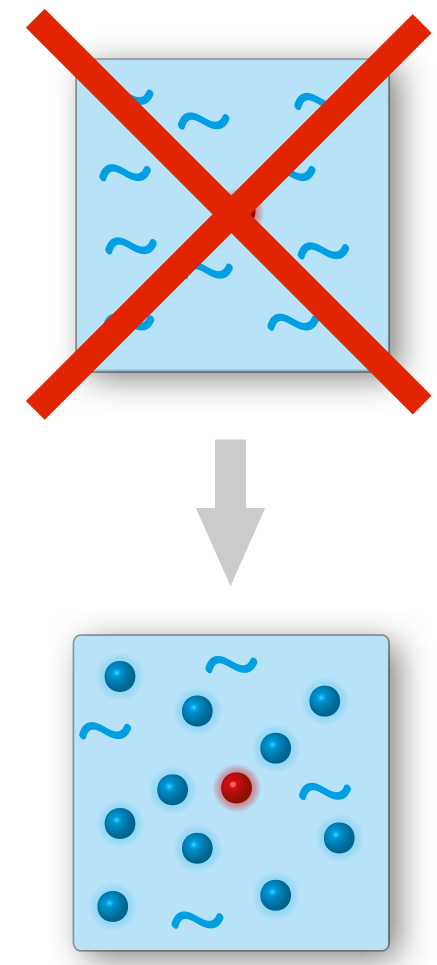
$$S_{\mathrm{eff}} = S_{\mathrm{kin}}^{\mathrm{Bos}} + S_{\mathrm{kin}}^{\mathrm{Imp}} + g_{IB} \int \left\{ \underbrace{\sqrt{n_B}\psi_x^*\psi_x(\phi_x + \phi_x^*) + n_B \psi_x^*\psi_x}_{\text{"Fröhlich terms"}} + \boxed{\psi_x^*\psi_x\phi_x^*\phi_x} \right\}$$

strong *effective* phonon-impurity interaction **comes at a prize**

1. microscopic attraction needed



$$g_{IB} \neq \frac{2\pi\hbar^2}{m_r} a_{IB}$$ ‘mean-field replacement’ invalid!



2. pairing fluctuations become relevant

→ MF approach & Fröhlich Hamiltonian becomes invalid

in RG language: *Froehlich*: weak coupling RG fixed point
cold atoms at Feshbach resonance: strong coupling RG fixed point

Our work: Bose polaron from a truly attractive model RATH, RS, PRA 88 (2013)

simple quantum field-theory approach

$$S = \int d^4x \varphi^*(x)\left(\partial_\tau - \frac{1}{2m_B}\nabla^2 - \mu_B\right)\varphi(x) + \psi^*(x)\left(\partial_\tau - \frac{1}{2m_I}\nabla^2 - \mu_I\right)\psi(x)$$

$$+\frac{g_B}{2}[\varphi^*(x)\varphi(x)]^2 + \tilde{g}_{IB}\psi^*(x)\psi(x)\varphi^*(x)\varphi(x)$$

bosons

impurity

attractive interaction

assume homogeneous, weakly interacting BEC

$$\varphi(\mathbf{x},t) = \rho_0^{1/2} + \phi(\mathbf{x},t)$$

mean-field **fluctuations**

Bogoliubov approximation for bosons: keep all terms up to quadratic in ϕ , ϕ^*

$$S_{\text{eff}} = \int_{\omega,\boldsymbol{p}} \left\{ \frac{1}{2}\begin{pmatrix}\phi_p^* \\ \phi_{-p}\end{pmatrix} \begin{pmatrix} -\left[G_\phi^{(0)}(-p)\right]^{-1} & g_{\phi\phi}\rho_0 \\ g_{\phi\phi}\rho_0 & -\left[G_\phi^{(0)}(p)\right]^{-1} \end{pmatrix} \begin{pmatrix}\phi_p \\ \phi_{-p}^*\end{pmatrix} + \psi_p^*\left(-i\omega + \frac{\boldsymbol{p}^2}{2m_\psi} - \mu_\psi\right)\psi_p \right\}$$

$$+\tilde{g}_{\phi\psi}\int_x \left[\psi_x^*\psi_x\phi_x^*\phi_x + \sqrt{\rho_0}\psi_x^*\psi_x(\phi_x + \phi_x^*) + \rho_0\psi_x^*\psi_x\right]$$

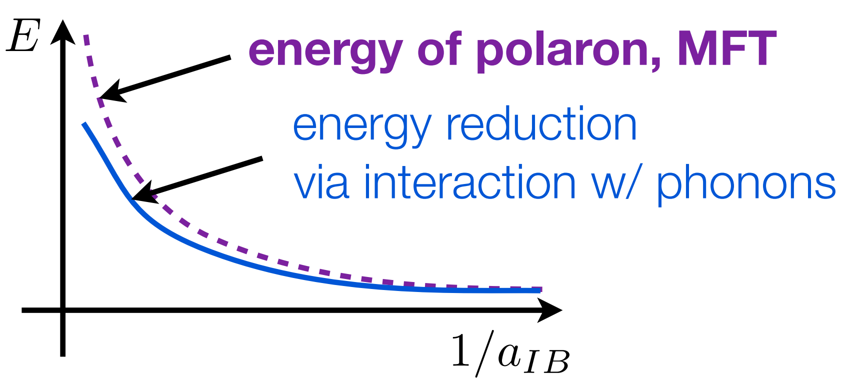
Unlike previous approaches, we keep **pairing fluctuations** "Fröhlich terms"

The Question.

What is the spectrum of the model?

do cold atoms forget underlying microscopic physics?

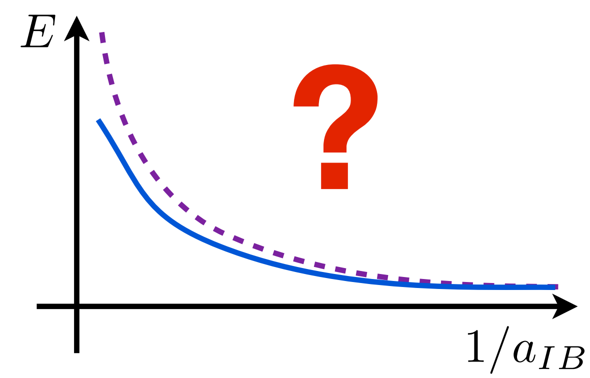
condensed matter



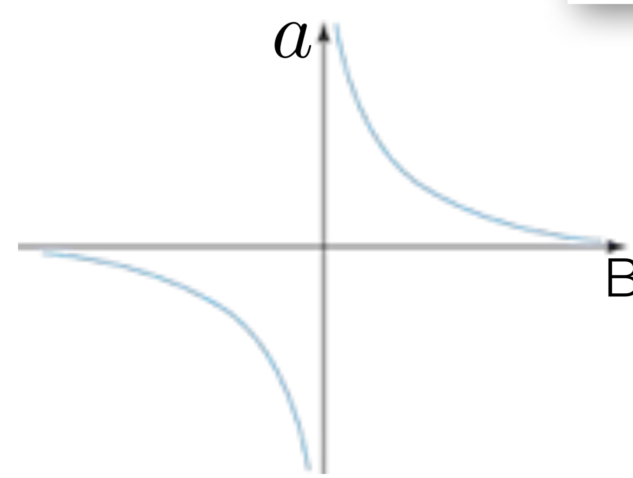
strict analogy

quantum simulation

cold atoms



exploiting
Feshbach resonance



Quantity to address this question: Spectral function

(gives access to radio-frequency response etc...)

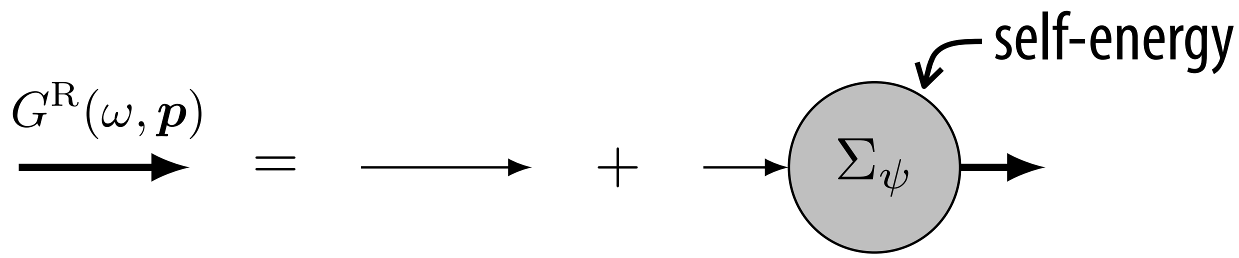
$$A_{\mathrm{pol}}(\omega, \boldsymbol{p}) = -2\,\mathrm{Im}\,G^{\mathrm{R}}(\omega, \boldsymbol{p})$$

T-matrix approximation

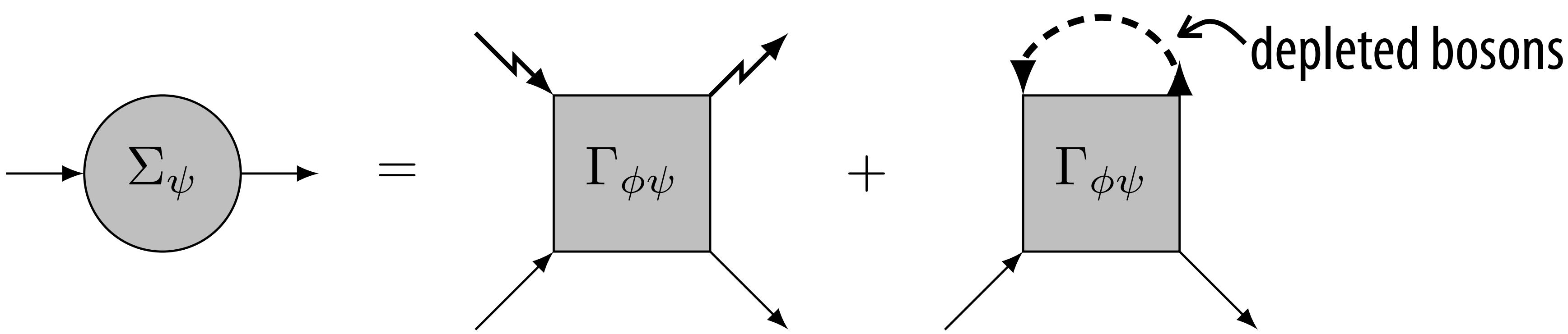
SEE ALSO FOR FERMIONS: **RS, ENSS, PRA 83 (2011)**

impurity spectral function $A_{\rm pol}(\omega,\boldsymbol{p}) = -2\,{\rm Im}\,G^{\rm R}(\omega,\boldsymbol{p})$ **from Dyson equation**

full Green's function:



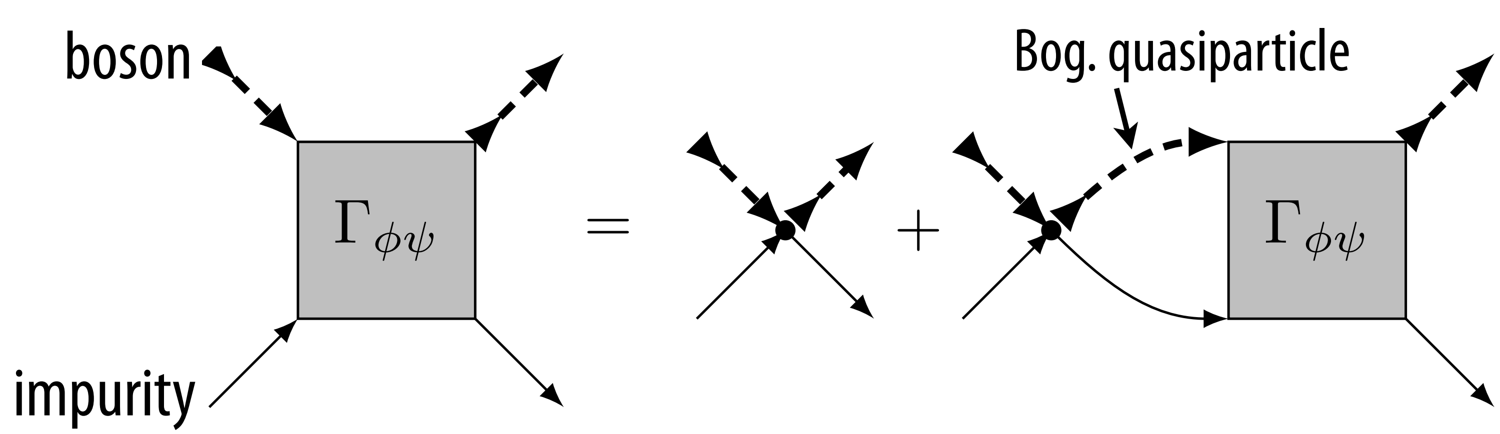
self-energy:



prerequisite: recover exact two-body solution [unlike previous works]

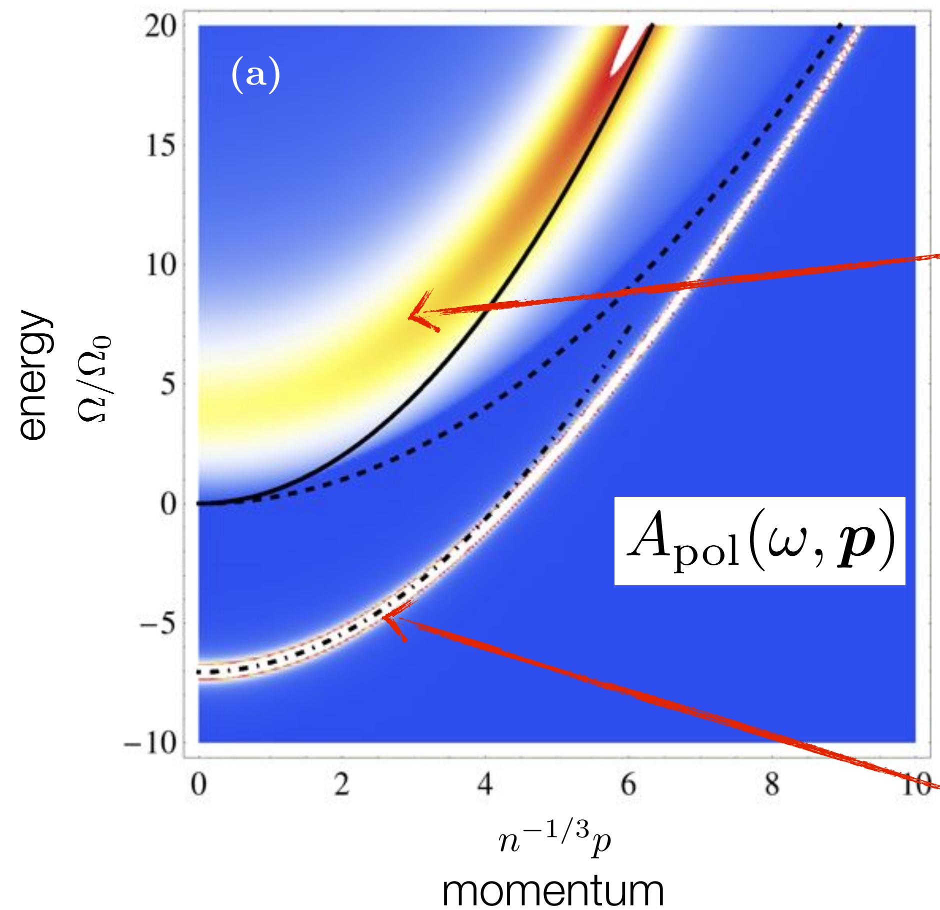
resummed perturbation theory

T-matrix equation:



Result for momentum resolved spectral function

$$n^{1/3}a_{IB}^{-1} = 1$$



Rath, RS, PRA 88 (2013)

unlike condmat: two coherent quasi-particle excitations!

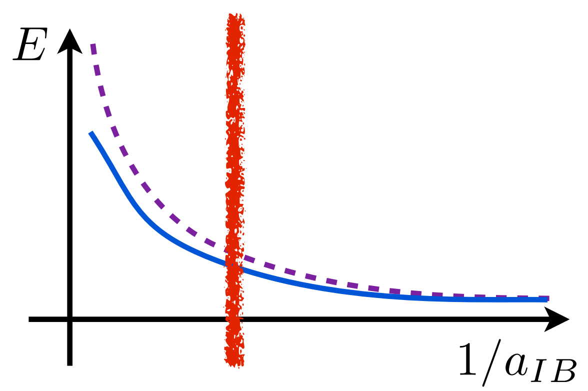
almost "standard" repulsive polaron $a > 0$

- at positive energy
- enhanced effective mass
- finite lifetime!
- largely reduced quasi-particle weight

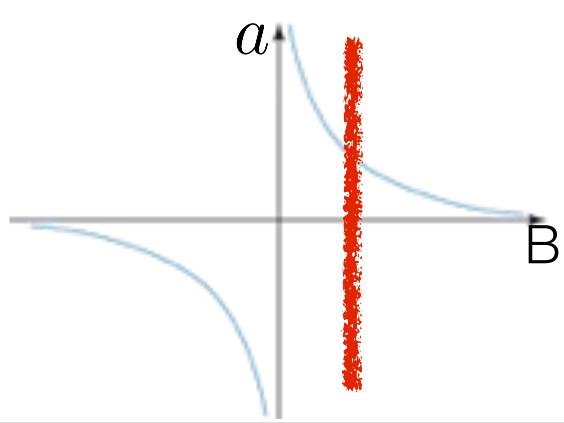
"new" attractive polaron $\forall\, a$

- actual ground state at negative energies!
- cannot be found in previous approaches
- interacts attractively with BEC
- enhanced effective mass

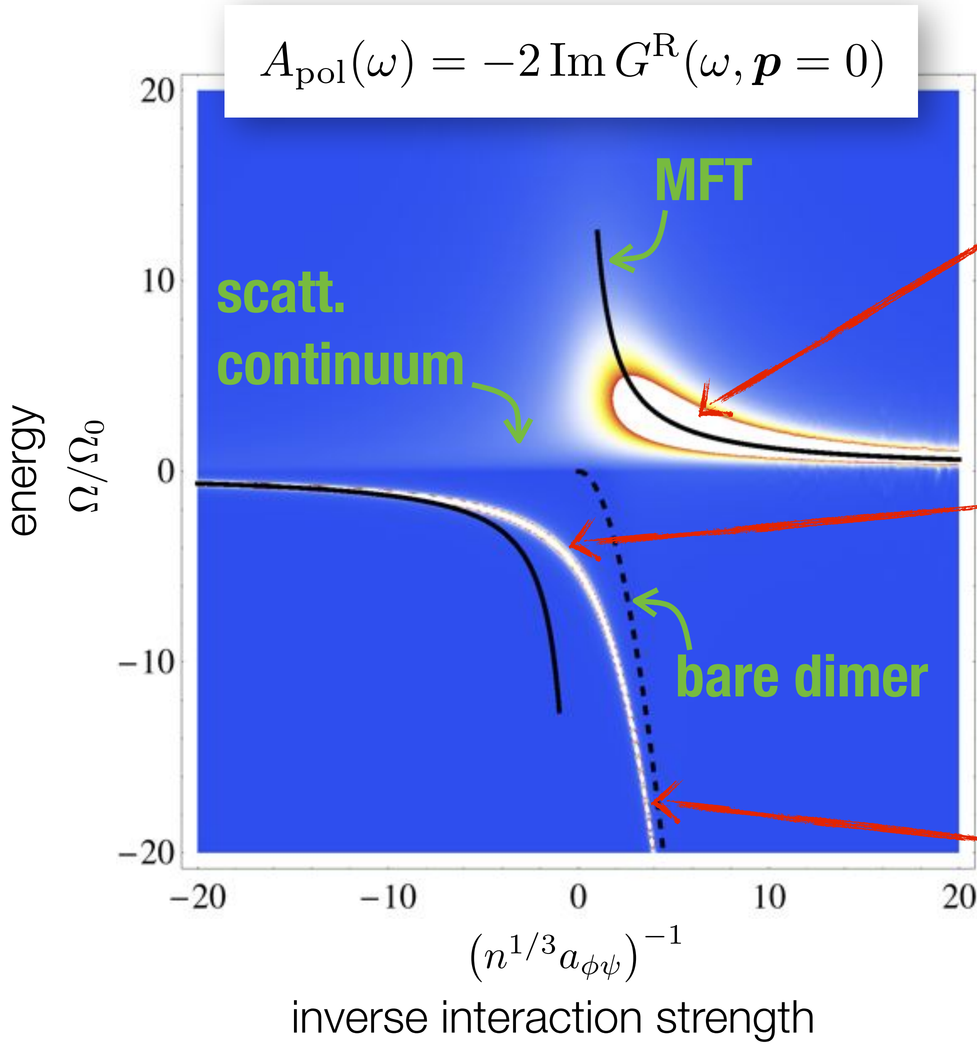
previous "quantum simulation proposals"

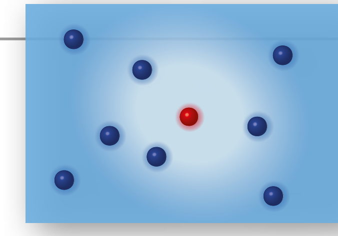


Feshbach resonance



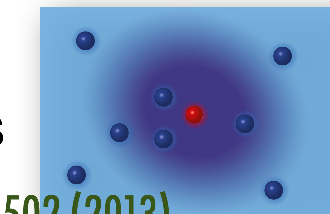
Energy spectrum for impurity at rest





repulsive polaron

- extremely unstable in strong-coupling regime! *[molecule formation]*
- self-localization challenging to observe



attractive polaron

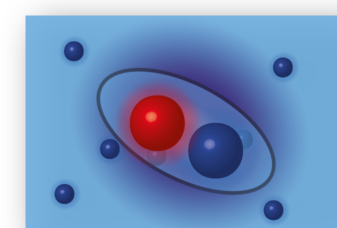
- stable ground state at all scattering lengths

OBSERVED AT WEAK COUPLING PFAU GROUP [STUTTGART]: BALEWSKI ET AL., NATURE 502 (2013)

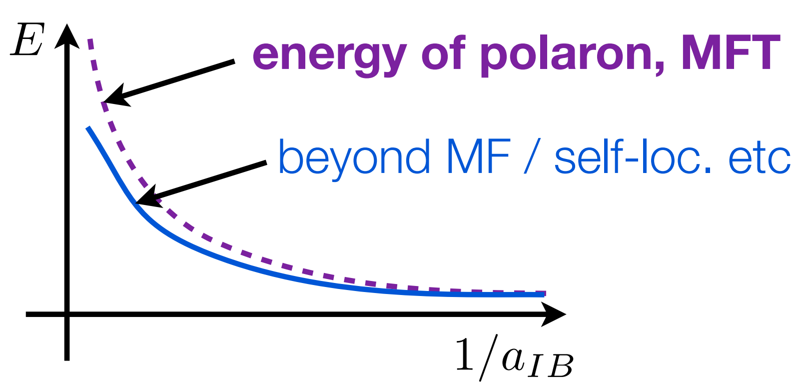
smooth **crossover to molecular state** - hybridization

- different from transition for Fermi polaron - RATH, RS, PRA 88 (2013)

DISCUSSED IN CONTEXT OF B/F MIXTURES BY MARCHETTI, ... PARISH, PRB 78 (2008)

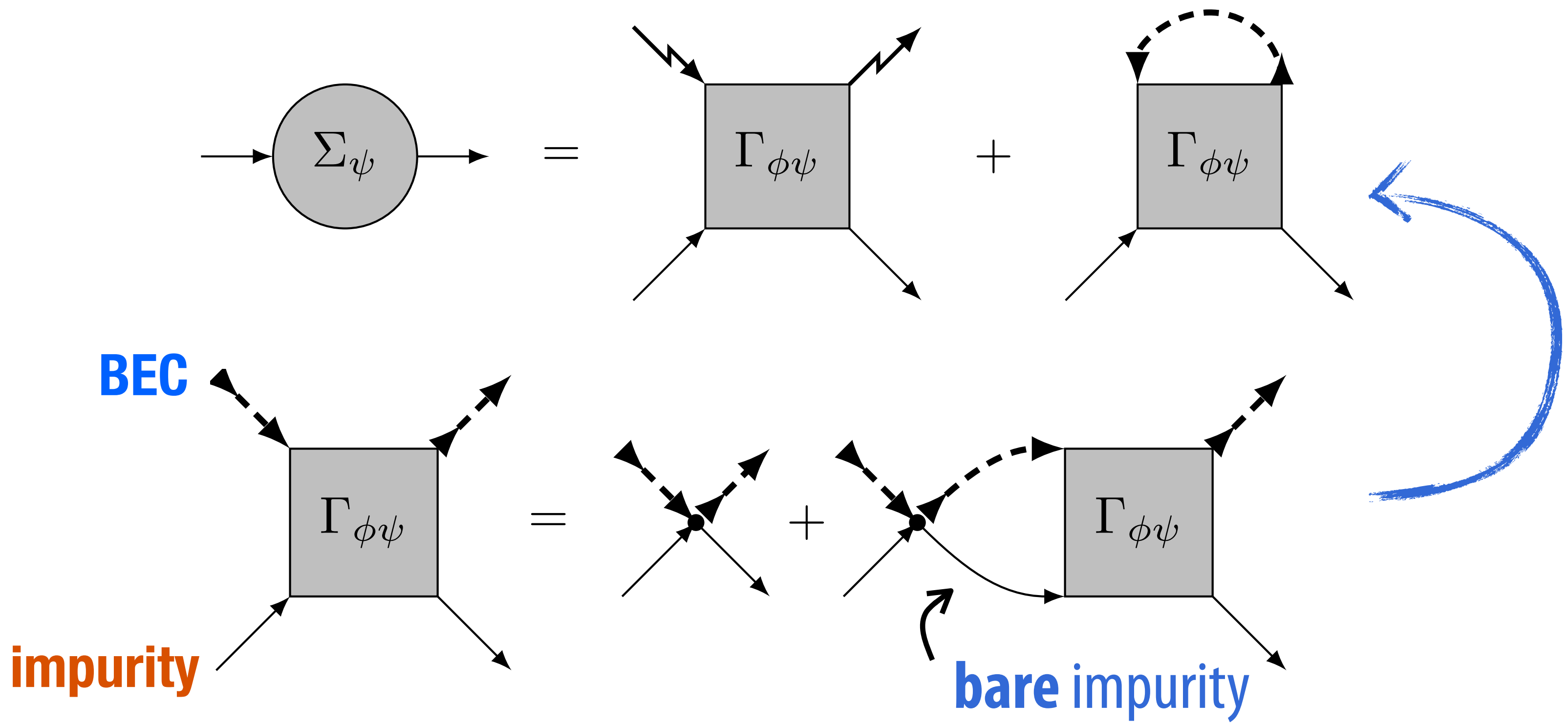


previous “quantum simulation proposals”



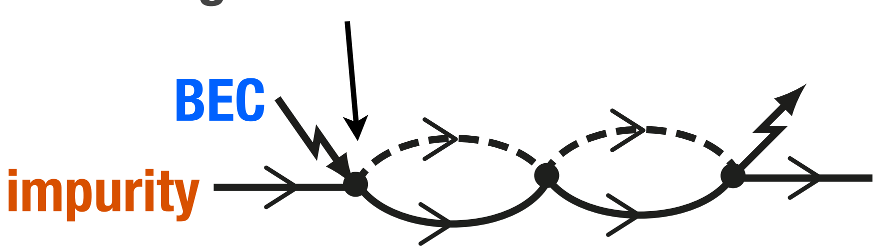
Self-consistent T-matrix

So far: Non-selfconsistent T-matrix approach



Non-self-consistent T-matrix

- **single boson taken out of condensate**



- **equivalent to simple variational wave function**

$$|\psi_0\rangle = \sqrt{Z}\hat{c}^\dagger_{\mathbf{0}}|\mathrm{BEC}\rangle + \sum_{\mathbf{k}} \mathcal{A}(\mathbf{k})\hat{c}^\dagger_{-\mathbf{k}}\hat{b}^\dagger_{\mathbf{k}}|\mathrm{BEC}\rangle$$

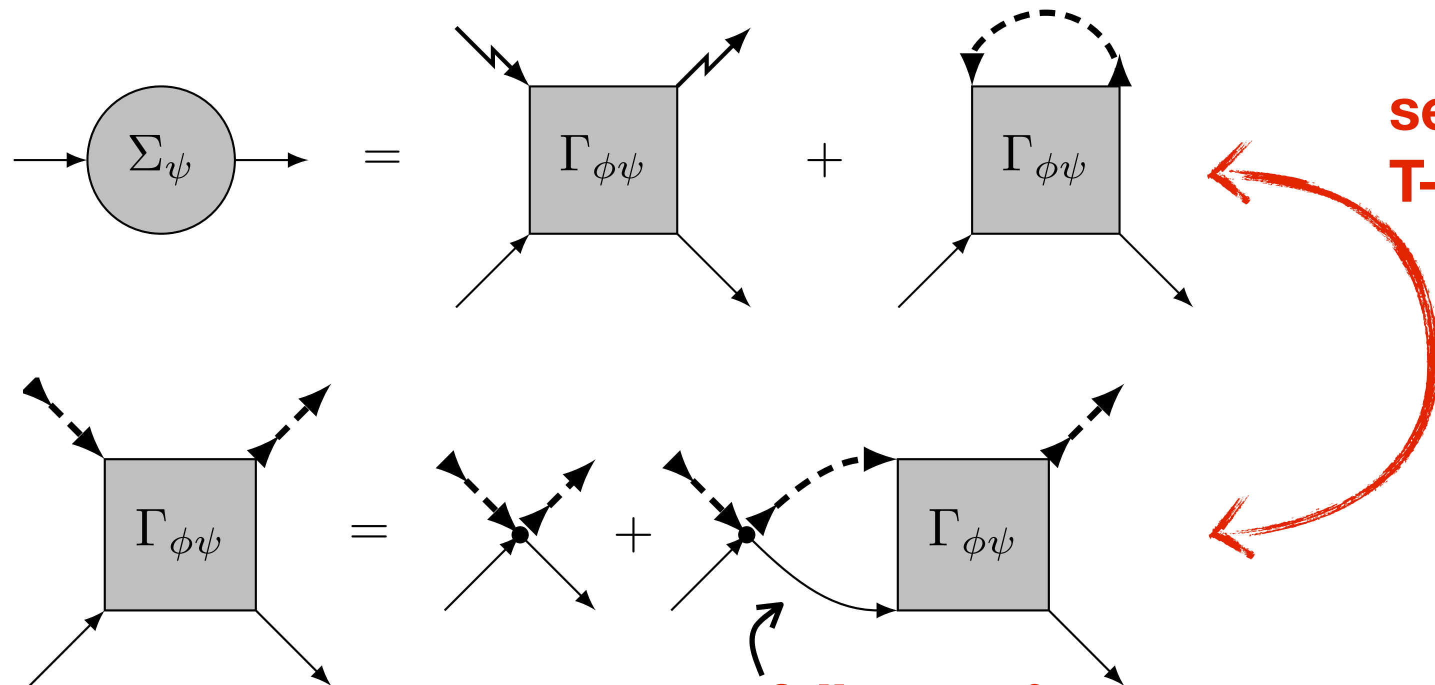
impurity

boson excited out of condensate

- captures very simple 'entanglement' between BEC and impurity

Self-consistent T-matrix

Selfconsistent T-matrix approach

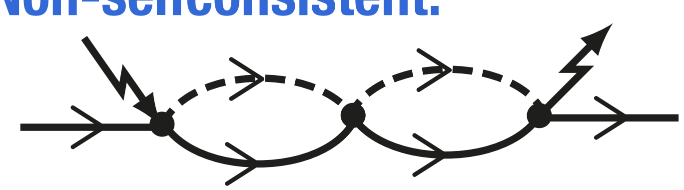


self-consistent T-matrix RATH, RS, PRA 88 (2013)

SEE ALSO FOR FERMIONS: RS, ENSS, PRA 83 (2011)

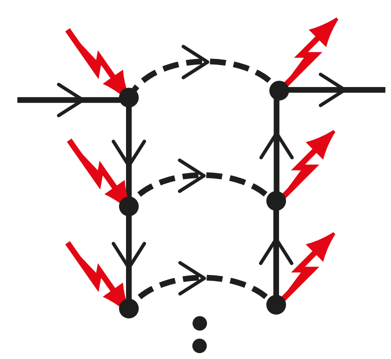
- solved numerically using algorithm developed for *functional renormalization group approach* for RG flow of full spectral functions RS, ENSS, PRA 83 (2011)
- accounts for infinitely many virtual excitations of bosons out of the coherent condensate state

Non-selfconsistent:



- **single boson taken out of condensate**

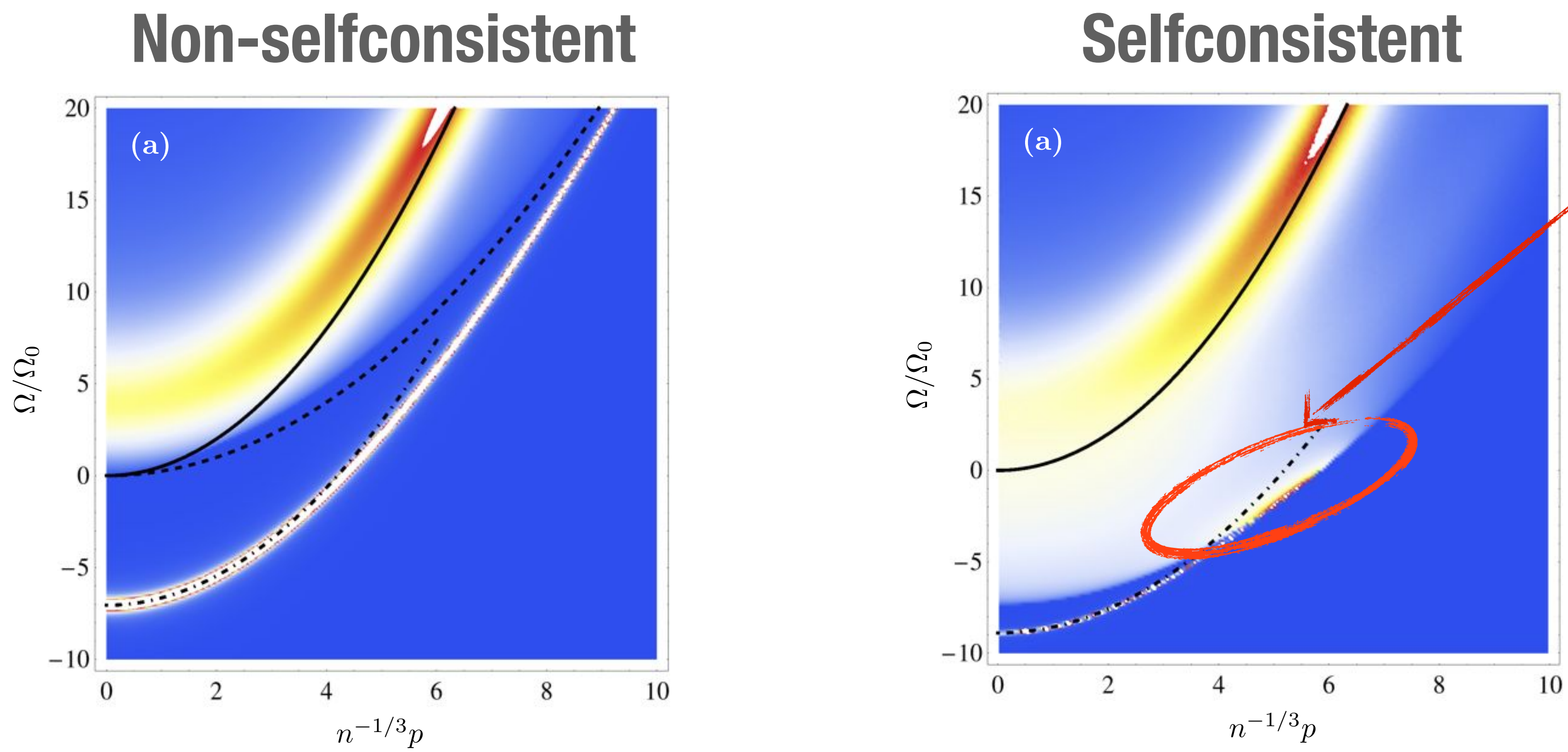
Selfconsistent:



- **infinite number of bosons taken out of condensate -** way beyond product wave functions for BEC

Self-consistent T-matrix - Results

Momentum resolved Spectral function

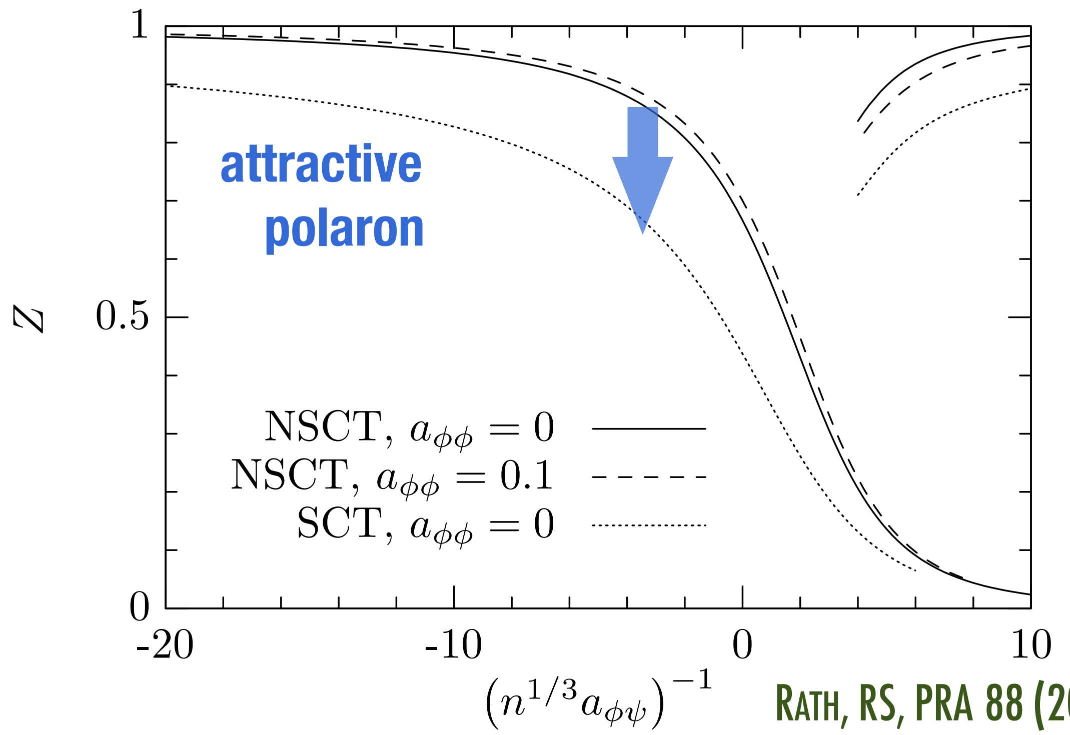


annihilation of fast attractive polaron at scattering threshold

↳ *momentum dependent Z-factor vanishes!*

SIMILAR TO FROEHLICH POLARON Z-FACTOR FROM DIAGMC, MISHCHENKO ET AL. PRB 62 (2000)

Suppression of quasi-particle weight



RATH, RS, PRA 88 (2013)

unlike for fermions:
simple variational wave functions not reliable for quantitative predictions for strong coupling Bose polaron

as for instance studied in LI AND DAS SARMA, ARXIV:1404.4054

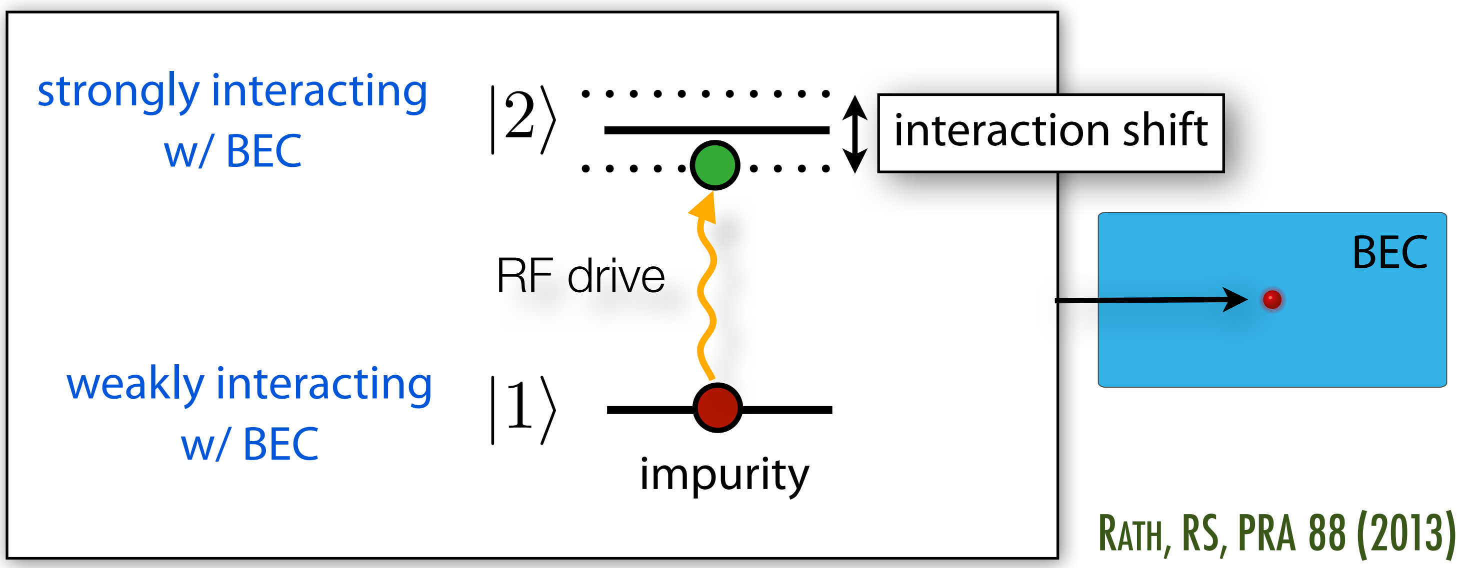
Proposal for experimental observation

E.G. $^{40}K/^{41}K$ MIXTURE AT B=543 G
SEE MIT GROUP: WU ET AL. PRA 84 (2012)

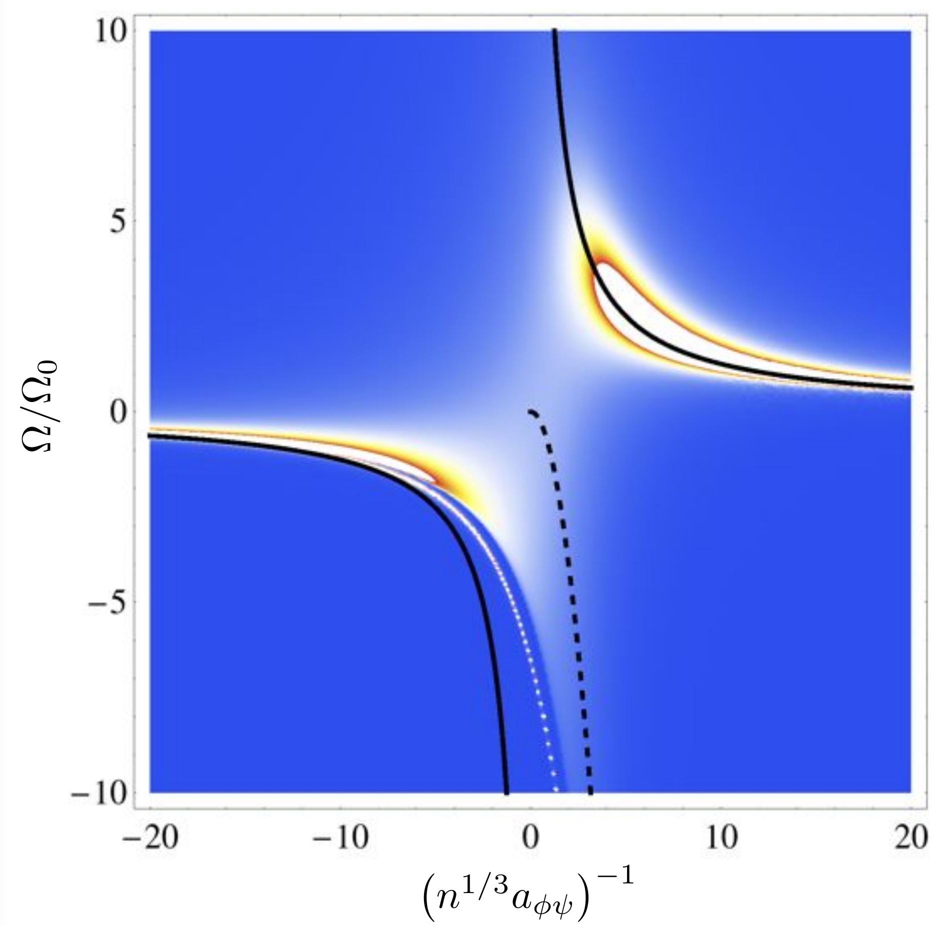
Challenge

- Efimov effect + statistics: Bose-Fermi mixtures unstable due to enhanced three-body recombination SEE E.G. RS, RATH, ZWERGER, EJB 85 (2012)
- possible BEC deformation due to large interactions

Resolution: Inverse RF spectroscopy



RATH, RS, PRA 88 (2013)



- maps out impurity spectral function
  - similar procedure proposed and used for fermions
- Efimov states off-resonant
- BEC deformation irrelevant as

$$\tau_{BEC} = \frac{\hbar}{\mu_B} = \frac{\hbar}{g_B n_B} \gg \tau_{\text{Rabi}}$$

- Quasiparticle weight measured via Rabi frequency shift

Outlook

1 **non-equilibrium physics**

- What happens on time scales longer than those of RF experiments?
- Our theory describes the polaron right after the drive to the final state
  Here the repulsive polaron is in a highly excited, non-equilibrium state

dynamical competition between molecule formation & Froehlich self-localization & bubble formation

↳ Repulsive Bose polaron as probe of nonequilibrium physics

2 **interplay few & many-body physics**

- Fate of Efimov physics in the realm of the polaron problem?

C.F.: ZINNER, EPL 101, 60009 (2013)

- Detailed study of impurity-molecule crossover

Thank you!