Towards new states of matter with atoms and photons

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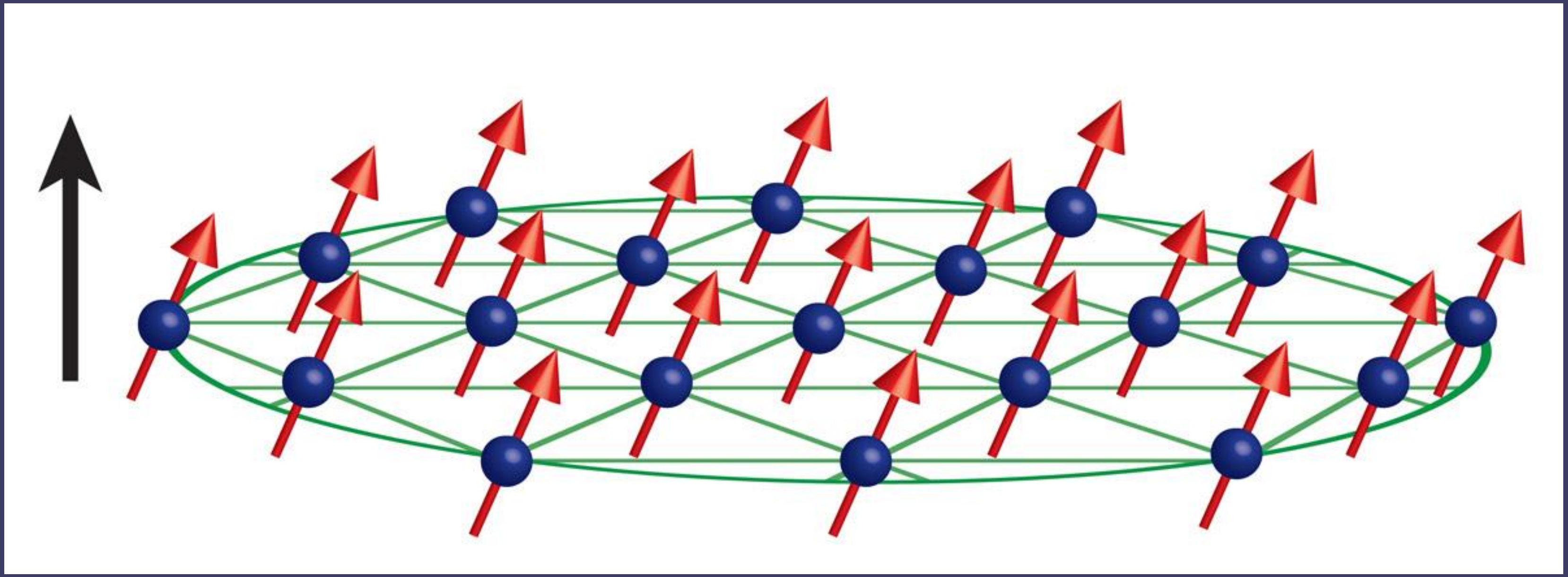
Aarhus “Cold atoms and beyond” 26/6-2014

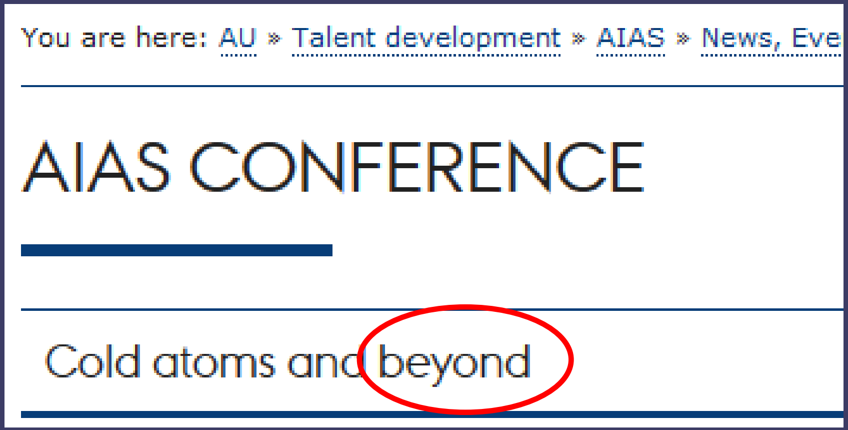
Motivation

- Optical lattices + control → quantum simulators.



- Hubbard models, spin models,…

Motivation



- High control → monitoring hybrid systems → many-body systems *beyond* condensed matter paradigm models.

Outline

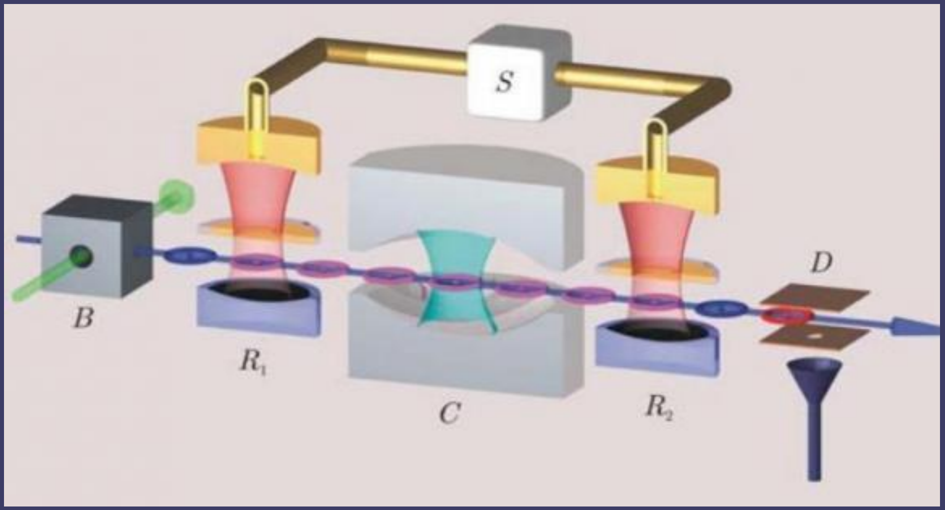
1. Cavity QED in five or so minutes.
2. Many-body cQED: (quantum) optical bistability
3. Collective phenomena, Dicke physics.
4. $SU(3)$ Dicke model.
5. And then…

Cavity Quantum Electrodynamics

Cavity QED

Jaynes-Cummings physics

- Cavity QED = coupling between few material (atomic) and few electromagnetic degrees of freedom.
- Cavity → atom-field coupling $g \sim {}^{1}/_{\sqrt{V}}$ (V effective mode volume).
- *Strong coupling regime* $g\sqrt{\bar{n}} \gg \gamma, \kappa$ (γ/κ atom/photon decay rates).



Classical atomic motion

Cavity QED

Jaynes-Cummings physics

- Jaynes-Cummings Hamiltonian

$$\hat{H}_{jc} = \omega\hat{a}^{+}\hat{a} + \frac{\Omega}{2}\hat{\sigma}_z + g(\hat{a}^{+}\hat{\sigma}^{-} + \hat{\sigma}^{+}\hat{a})$$

Field energy　　Atomic energy　　Interaction energy

- *Dressed states* (polaritons)

$$|\psi_{n+}\rangle = \cos\theta\,|e,n\rangle + \sin\theta\,|g,n+1\rangle,$$

$$|\psi_{n-}\rangle = \cos\theta\,|e,n\rangle - \sin\theta\,|g,n+1\rangle,$$

$$E_{n\pm} = \omega n \pm \sqrt{\frac{\Delta^2}{4} + g^2(n+1)}, \qquad \tan 2\theta = 2g\sqrt{n+1}/\Delta,$$

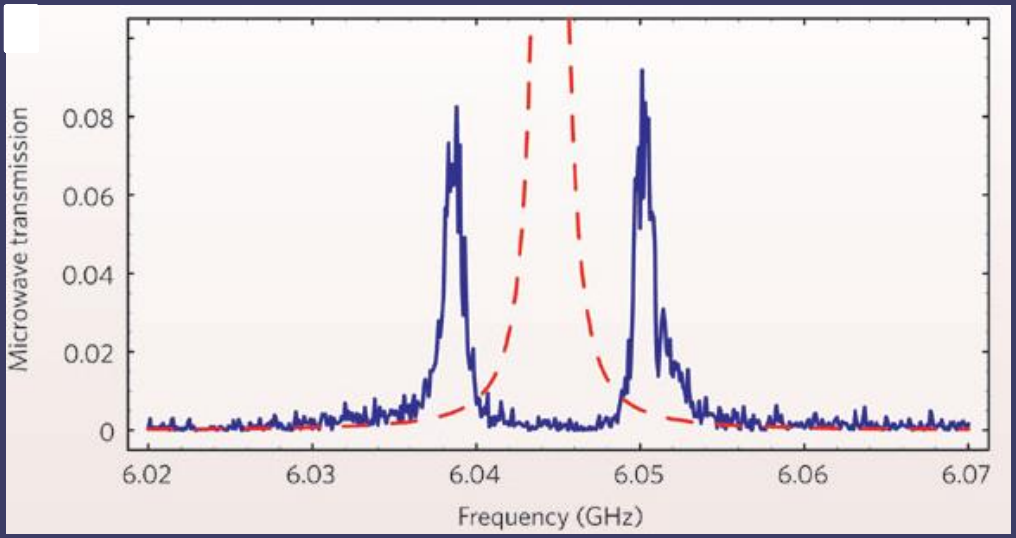
$$\Delta = \Omega - \omega.$$

Haroche & Raimond, *Exploting the Quantum* (OUP).

Cavity QED

Jaynes-Cummings physics

- *Vacuum Rabi splitting*, $E_{0\pm} = \pm g \quad (\Delta = 0)$

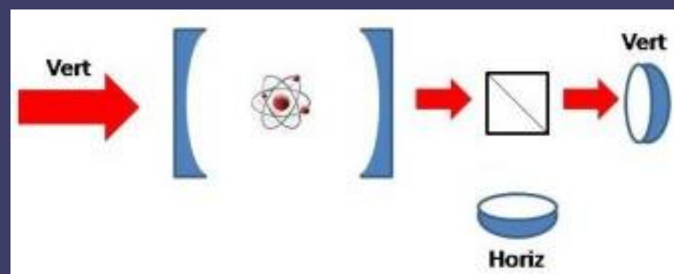


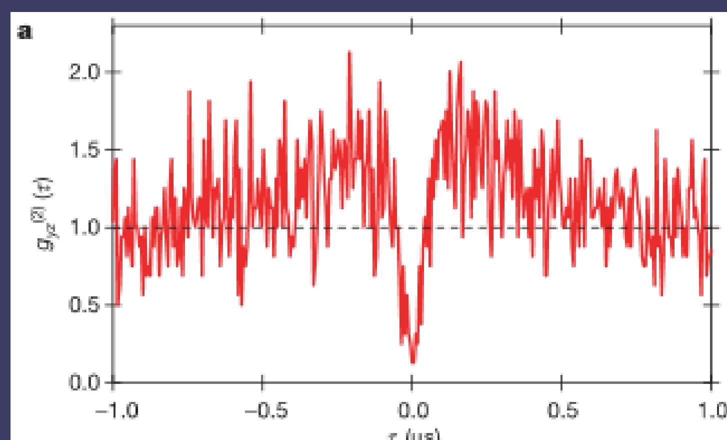
Schoelkopf & Girvin, *Nature* **451** (2008).

Cavity QED

Jaynes-Cummings physics

- *Photon blockade*



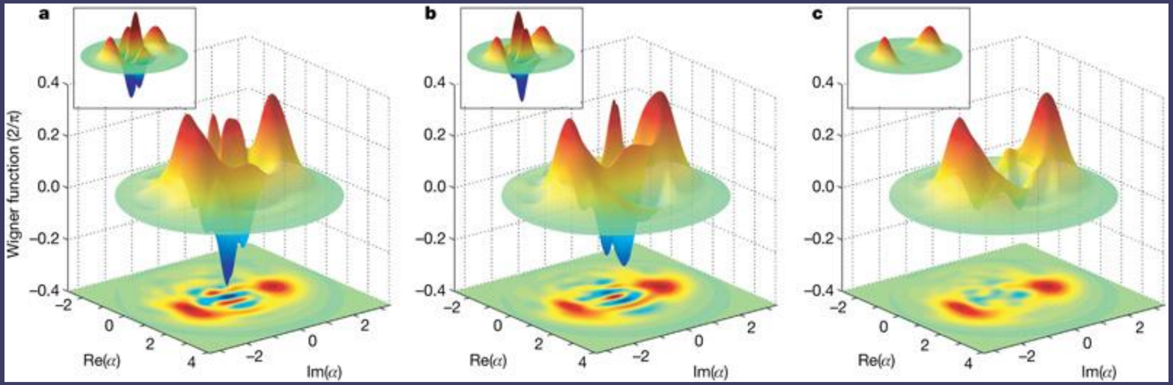


Kimble, *Nature* **436** (2005).

Cavity QED

Jaynes-Cummings physics

- Atom-atom, atom-field, field-field entanglement.
- Logic gates.
- ”Cats”.
- Tomography.



- Quantum-classical correspondence.
- Zeno and measurement phenomena.
- Field quantization.
- …

Haroche & Raimond, *Exploting the Quantum* (OUP).

Cavity QED

Dicke physics

- Dicke 1954: How does *N* atoms radiate?

$$\hat{H}_d = \omega\hat{a}^{+}\hat{a} + \sum_{i=1}^{N}\frac{\Omega}{2}\hat{\sigma}_i^z + g(\hat{a}^{+} + \hat{a})\hat{\sigma}_i^x \equiv \omega\hat{a}^{+}\hat{a} + \frac{\Omega}{2}\hat{S}_z + g(\hat{a}^{+} + \hat{a})\hat{S}_x$$

- *Dicke states* $\hat{S}_z|S,m\rangle = m|S,m\rangle$ (maximal spin sector $S = N/2$).

$\sum_{\psi}\left|\langle\psi|\hat{H}_d|S,S,0\rangle\right|^2 \sim N^2$ *Superradiance!*

- Enhanced radiation by a factor N!

Dicke, *Phys. Rev.* **93** (1954).

Cavity QED

Dicke physics

- Thermodynamic limit ($g \to g/\sqrt{N}$). Z_2-parity symmetry breaking.

$$g_c = \sqrt{\omega\Omega}/2 \qquad \begin{cases} Normal\ phase: & \langle \hat{a} \rangle = 0, \ \langle S_z \rangle = -N/2 \\ Superradiant\ phase: & \langle \hat{a} \rangle \neq 0, \ \langle S_z \rangle > -N/2 \end{cases}$$

- Classical PT (*Ising* type), survives also at $T = 0$ ('quantum' PT).
- *No-go theorem.* Minimal coupling Hamiltonian, assume ground state atoms, no dipole-dipole interaction → Dicke PT forbidden (Rzaźewski, *Phys. Rev. Lett*. **35**, (1975)).
- Large atom-field coupling by "driving" → openness → PT?

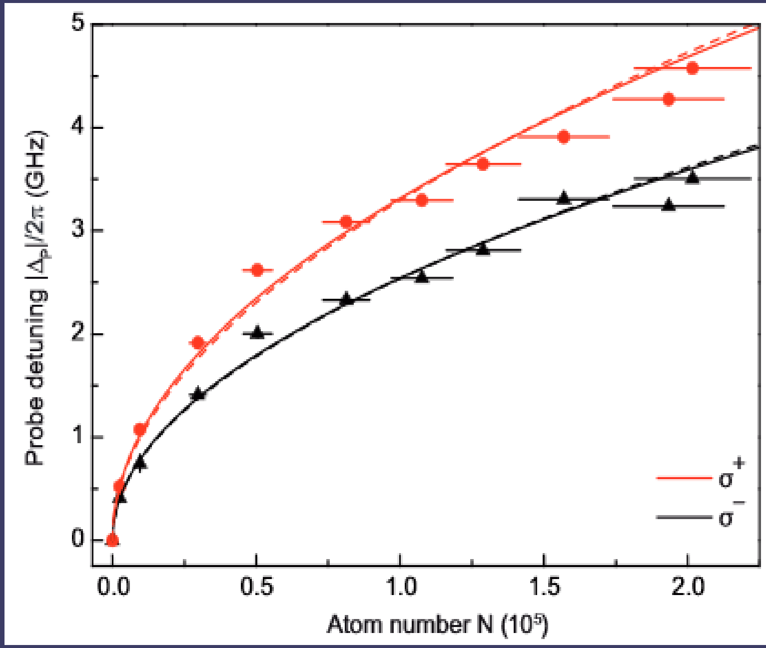
Wang & Hioe, *Phys. Rev. A* **7** (1973).

Many-body cQED

Many-body cQED

Optical bistability

- Thermal atomic gases had been loaded into resonators, next step a BEC.
- 2007; Stamper-Kurn, Reichel, Esslinger. (no strict proof for BEC at this stage).
- Vacuum Rabi splitting $\sim\sqrt{N}$.



Brennecke, *Nature* **450** (2007) .

Many-body cQED

Optical bistability

- Low temperature $\rightarrow$ coupling to atomic motion.
- Hamiltonian in dispersive regime ($|\Delta| \gg g\sqrt{\bar{n}}$, $U_0 = g^2/\Delta$)

$$\hat{H} = \int dx\, \hat{\psi}^+(x)\left[-\frac{d^2}{dx^2} + U_0 \cos^2(x)\hat{a}^+\hat{a}\right]\hat{\psi}(x) - \Delta_c \hat{a}^+\hat{a} - i\eta(\hat{a} - \hat{a}^+).$$

- Rotating frame ($\Delta_c = \omega - \omega_p$), standing wave of resonator, longitudinal cavity pumping (amplitude η, frequency ω_p), neglecting atom-atom interaction.
- Steady state photon number

$$n_s = \frac{\eta^2}{\kappa^2 + \left(\Delta_c - U_0 \int dx |\psi(x)|^2 \cos^2(kx)\right)^2}.$$

Ritsch, *Rev. Mod. Phys*. **85** (2013) .

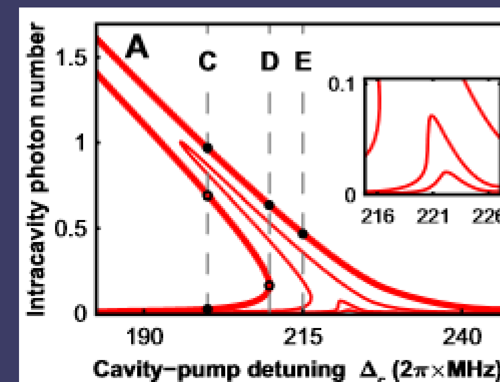
Many-body cQED

Optical bistability

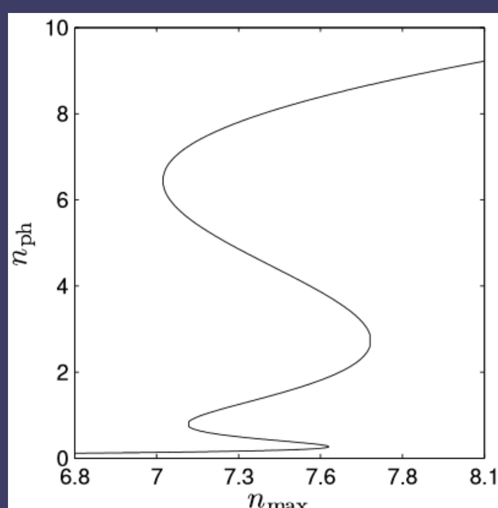
- Lowest vibrational modes of the BEC:

$$\hat{H} = \omega_{rec}\hat{c}^{+}\hat{c} + (-\Delta_c + g(\hat{c} + \hat{c}^{+}))\hat{a}^{+}\hat{a} - i\eta(\hat{a} - \hat{a}^{+}).$$

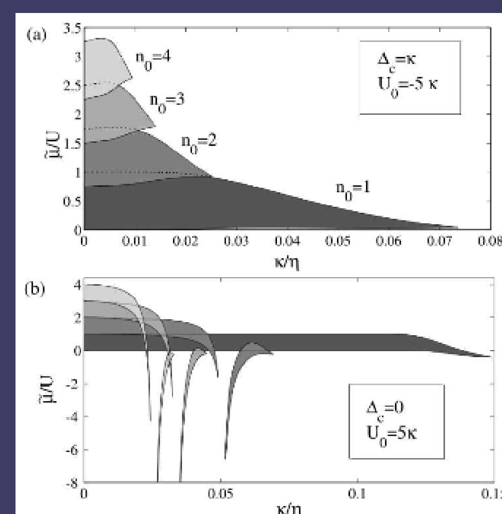
- Paradigm optomechanical Hamiltonian.
- Multi-stability and strong coupling regime



Brennecke, *Science* **322** (2008) .



Venkatesh, *Phys. Rev. A* **83** (2011) .

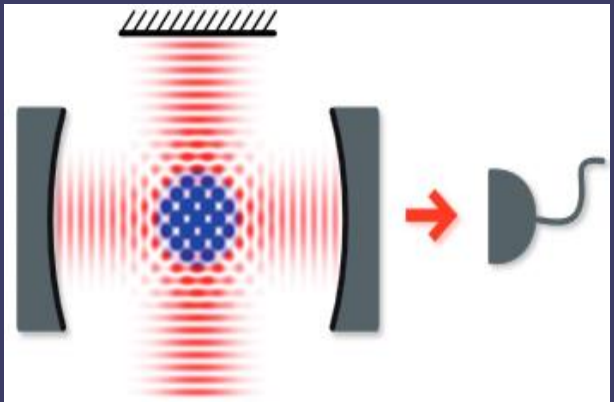


Larson, *Phys. Rev. Lett*. **100** (2008) .

Many-body cQED

Dicke physics

- Transverse pumping: atoms mediate scattering of photons from pump to cavity.
- Expand in lowest vibrational modes → Effective model = Dicke Hamiltonian in the *Schwinger representation*.



- Effective potential

$$V_{eff}(x,z) = V_0 \cos^2(z) + U_0 |\alpha|^2 \cos^2(x) + W_0(\alpha + \alpha^*) \cos(x) \cos(z)$$

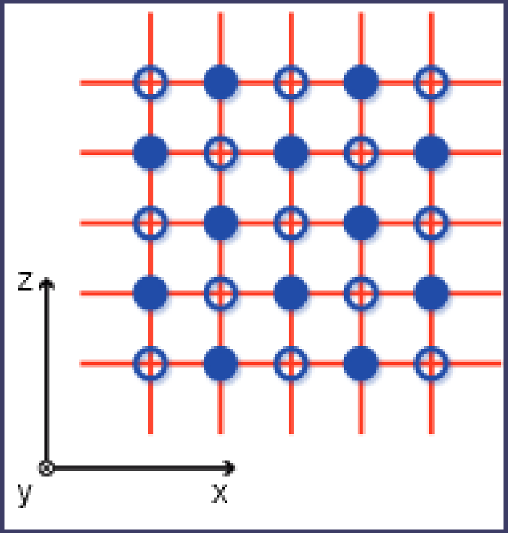
- Field phase $\mathrm{angle}(\alpha + \alpha^*) = 0,\ \pi \ \rightarrow Z_2$ symmetry breaking of Dicke.
- Condensates in a ”checkerboard” supersolid phase: populates every second site.

Nagy, *Phys. Rev. Lett*. **104** (2010).

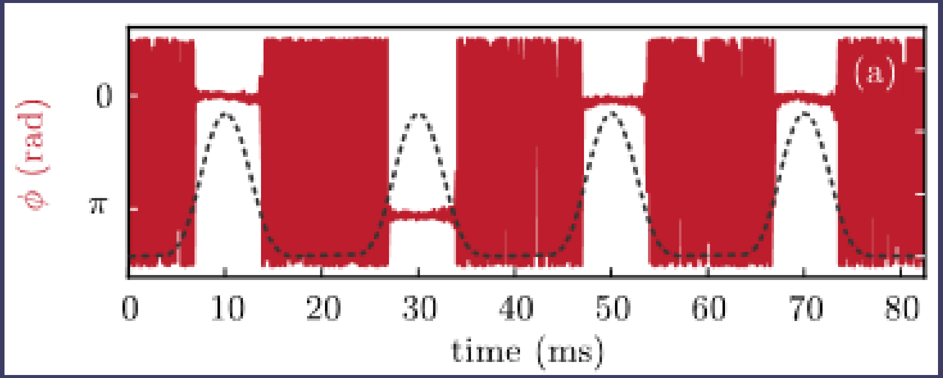
Many-body cQED

Dicke physics

- Experimental realizations:



Schematic picture of supersolid states (*self-organization* transition).



Experimental measurement of field phase.

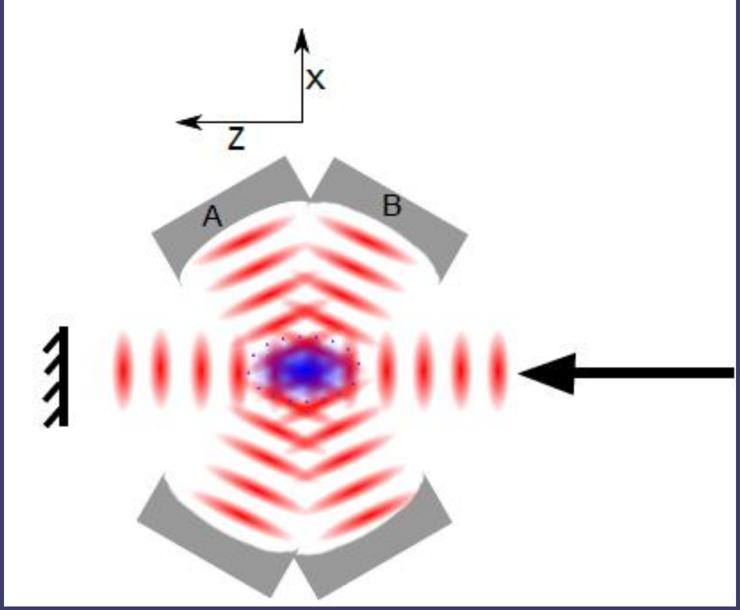
Baumann, *Nature* **464** (2010) , *Phys. Rev.L ett*. **107** (2011).

$SU(3)$ Dicke model

Many-body cQED

Beyond regular Dicke physics

- Esslinger group, new setup 1:

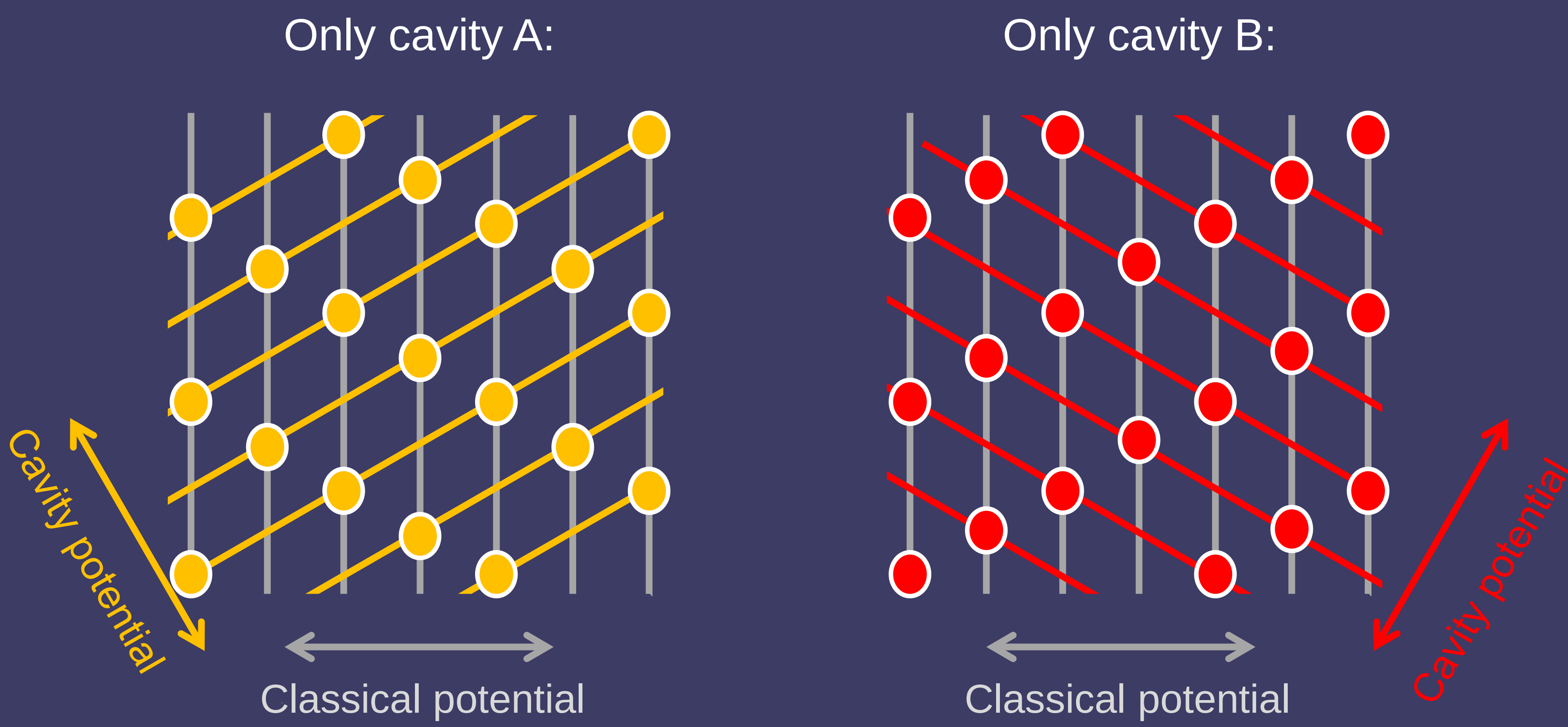


Schematic picture of the 'double-cavity setup'.

Venkatesh & Larson, underway

Many-body cQED

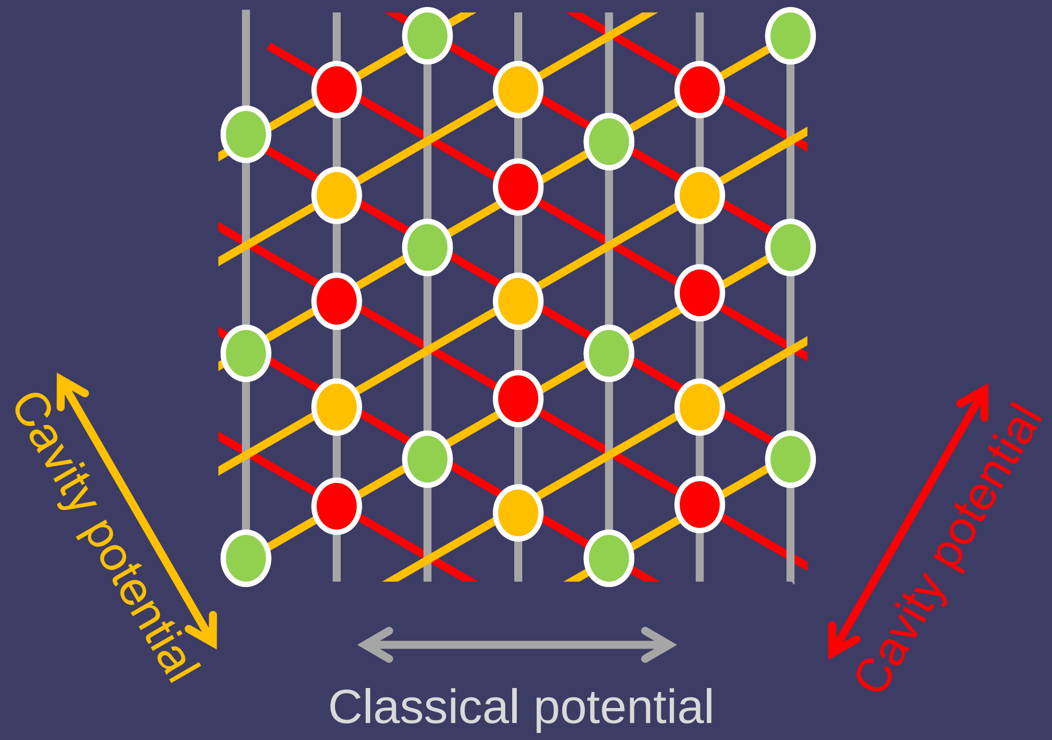
Beyond regular Dicke physics



Many-body cQED

Beyond regular Dicke physics

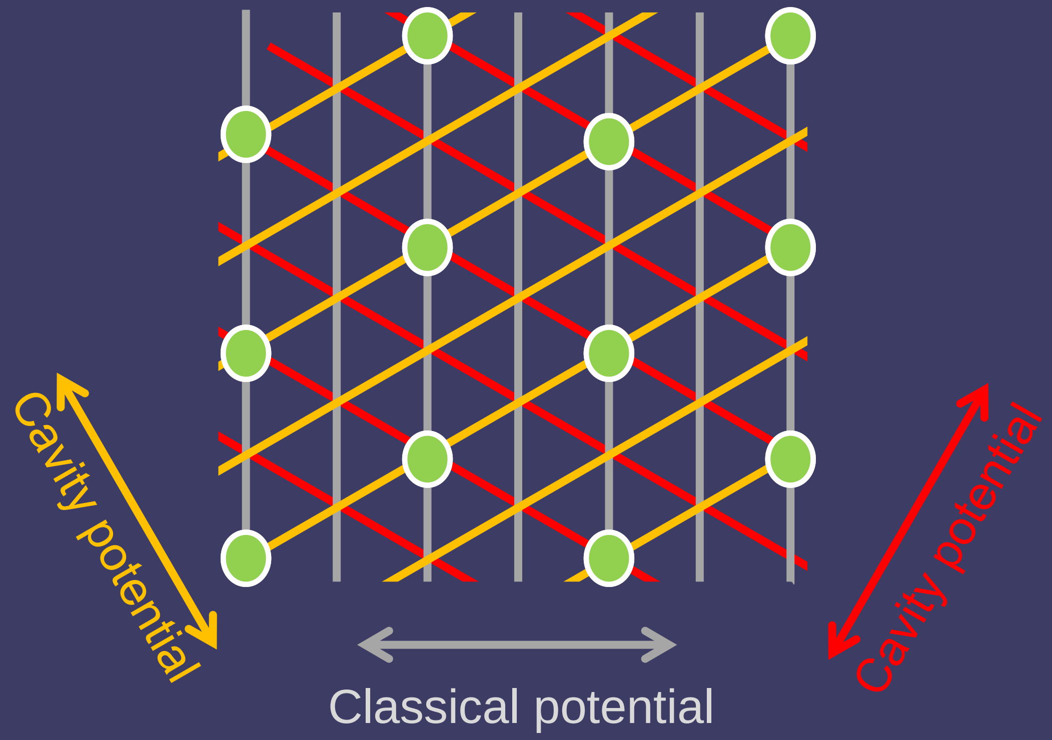
Cavity A + B:



Many-body cQED

Beyond regular Dicke physics

Cavity A + B:



No classical frustration!

$Z_2 \times Z_2$ symmetry.

Venkatesh & Larson, underway

Many-body cQED

Beyond regular Dicke physics

- Physics beyond mean-field.
- Identify the low energy vibrational states

$$\hat{\Psi}(x,z) = \hat{c}_0\psi_0(x,z) + \hat{c}_1\psi_1(x,z) + \hat{c}_2\psi_2(x,z),$$

$$\psi_0(x,z) = \frac{1}{\sqrt{N}}, \quad \psi_{1,2}(x,z) = \frac{2}{\sqrt{N}}\cos\left(\frac{\sqrt{3}}{2}x \pm \frac{1}{2}z\right)\cos(z),$$

- Generalized Schwinger representation

$$\hat{H}_{su3} = \omega_A \hat{a}^+\hat{a} + \omega_B b^+\hat{b} + \Omega\hat{\Lambda}_8 + \frac{g_A}{\sqrt{N}}(\hat{a} + \hat{a}^+)\hat{\Lambda}_4 + \frac{g_B}{\sqrt{N}}(\hat{b} + b^+)\hat{\Lambda}_6.$$

- $\hat{\Lambda}_i,\ i = 1, \dots, 8$ $SU(3)$ algebra.
- $\hat{\Lambda}_4, \hat{\Lambda}_6$ and $\hat{\Lambda}_8$ ***not*** an $SU(2)$ subgroup.

Venkatesh & Larson, underway

Many-body cQED

Beyond regular Dicke physics

- Symmetries:

1. $Z_2 \times Z_2$ generalized Dicke parity symmety

$$\hat{\Pi}_A = \exp\left[i\pi\left(\hat{n}_A + \hat{\Lambda}_3/2 + \sqrt{3}\,\hat{\Lambda}_8/2\right)\right],$$

$$\hat{\Pi}_B = \exp\left[i\pi\left(\hat{n}_B - \hat{\Lambda}_3/2 + \sqrt{3}\,\hat{\Lambda}_8/6\right)\right].$$

2. If parameters $X_A = X_B$ an 'emergent' continuous $U(1)$ symmetry

$$\hat{U}(\theta) = \exp\left[-i\theta\left(\hat{a}^{+}\hat{b} - \hat{b}^{+}\hat{a}\right) - i\theta\hat{\Lambda}_2\right].$$

Venkatesh & Larson, underway

Many-body cQED

Beyond regular Dicke physics

- Mean-field of $SU(3)$ Dicke

$$\dot{\Lambda}_\alpha = \cdots\,, \quad \alpha = 1,2,\ldots,8,$$

$$\dot{a} = \cdots, \qquad \dot{b} = \cdots.$$

- Steady state solutions ($U(1)$ symmetry $X_A = X_B$):

$$\frac{\Lambda_4}{N} = \cos\theta/2\,\sqrt{1 - {g_c}^4/g} \qquad \text{("Dipole 01")},$$

$$\frac{\Lambda_6}{N} = \sin\theta/2\,\sqrt{1 - {g_c}^4/g} \qquad \text{("Dipole 02")},$$

$$\frac{\Lambda_1}{N} = \frac{\sin\theta}{2}\left(1 - {g_c}^2/g\right) \qquad \text{("Dipole 12")},$$

$$\frac{\Lambda_3}{N} = \frac{\cos\theta}{2}\left(1 - {g_c}^2/g\right) \qquad \text{("Inversion 12")},$$

$$\frac{a+a^*}{\sqrt{N}} = -\frac{2\omega g}{\kappa^2+\omega^2}\cos\theta/2\,\sqrt{1 - {g_c}^4/g^4} \qquad \text{(Quadrature A)},$$

$$\frac{b+b^*}{\sqrt{N}} = -\frac{2\omega g}{\kappa^2+\omega^2}\sin\theta/2\,\sqrt{1 - {g_c}^4/g^4} \qquad \text{(Quadrature B)}.$$

Venkatesh & Larson, underway

Many-body cQED

Beyond regular Dicke physics

- Critical coupling $g_c = \frac{1}{2}\sqrt{\frac{\Omega(\kappa^2+\omega^2)}{\omega}}$, cavity losses κ.
- $x_A, x_B \sim |1 - \frac{g_c}{g}|^{1/2} \rightarrow \beta = -1/2$ as in ”open” $SU(2)$ Dicke model.
- For the $Z_2 \times Z_2$ identical critical coupling g_c and exponent β.
- Breaking of $Z_2 \times Z_2 \rightarrow$ one cavity empty in superradiant phase.
- Breaking of $U(1) \rightarrow$ both civities populated, amplitude set by θ.

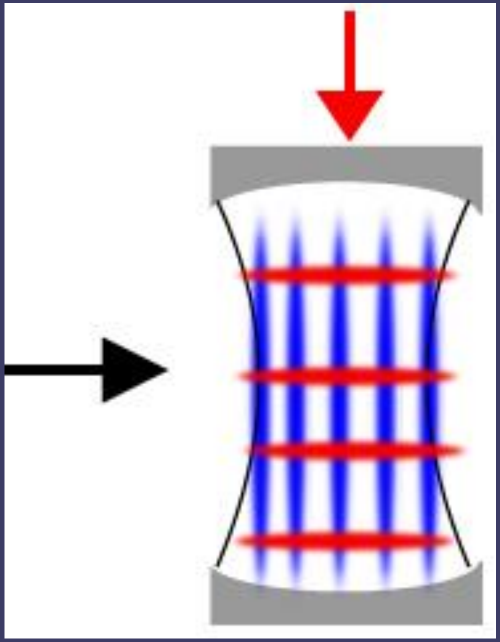
Venkatesh & Larson, underway

What's next?

Long range interaction

Glassy states?

- Esslinger group, new setup 2:



Atoms in (classical) 2D optical lattice → 1D tubes. Setup confined in a resonator.

“Tubes” independently interact with the same cavity mode.

~1500 tubes, 10-100 atoms/tube.

Long range interaction

Glassy states?

Atom-atom interaction

- *N*-channel Dicke realization

$$\hat{H}_{Nd} = \omega \hat{a}^{+}\hat{a} + \Omega \sum_{i=1}^{N} \hat{S}^{(i)}{}_{z} + g_i(\hat{a} + \hat{a}^{+})\hat{S}^{(i)}{}_{x} + U\hat{S}^{(i)}{}_{z}\hat{S}^{(i)}{}_{z}.$$

- Trace out boson field

$$\hat{H}_{rLMG} = \sum_{i=1}^{N} \hat{S}^{(i)}{}_{z} + U\left(\hat{S}^{(i)}{}_{z}\right)^{2} + \sum_{i,j} g_{ij}\hat{S}^{(i)}{}_{x}\hat{S}^{(j)}{}_{x}.$$

- *N*-channel (quasi random) *Lipkin-Meshkov-Glick model*.
- g_{ij} Gaussian distributed $\overset{?}{\rightarrow}$ glassy states (Strack & Sachdev, *Phys. Rev. Lett*. **107**, (2011); Buchhold, *Phys. Rev. A*. **87**, (2013)).

Long range interaction

Localization

- Optical lattice + cavity field (standing wave).
- Incommensurate lattices (cavity field weak), mean-field (atomic part)

$$\partial_t \varphi_i = -J(\varphi_{i-1} + \varphi_{i+1}) + \mu_i \hat{a}^+ \hat{a} \varphi_i,$$
$$\partial_t \hat{a} = -i\Delta\hat{a} - i\sum_i \mu_i {\varphi^*}_i \varphi_i \, \hat{a} + \eta.$$

- $\partial_t \varphi_i = -J(\varphi_{i-1} + \varphi_{i+1}) + \mu_i \varphi_i$, onsite disorder → localization (Kramer & MacKinnon, *Rep. Prog. Phys*. **56**, (1993)) .
- $\partial_t \varphi_i = -J(\varphi_{i-1} + \varphi_{i+1}) + \mu_i \varphi_i + |\varphi_i|^2 \varphi_i$, weak nonlinearity → localization lost at 'large times' (Pikovsky & Shepelyansky, *Phys. Rev. Lett*. **100**, (2008)).
- $\partial_t \varphi_i = -J(\varphi_{i-1} + \varphi_{i+1}) + \mu_i f\left(\sum_i \mu_i |\varphi_i|^2\right) \varphi_i$, localization?


Thanks!