The angular momentum problem in ^{3}He and topological superconductors: Using cold atoms to solve a 40 year old problem

Ed Taylor
McMaster University

Cold atoms and beyond, AIAS, June 27 2014

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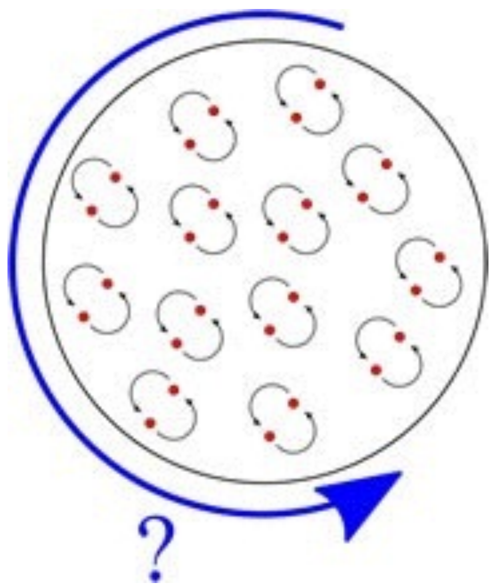
The angular momentum problem



Oshikawa. Tokyo

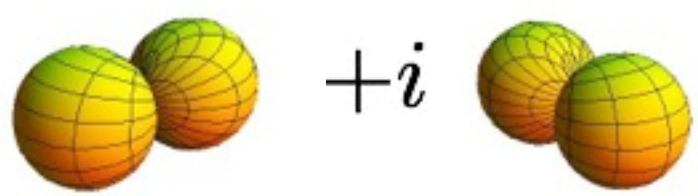
- Originally arose in theory studies of ^{3}**He** in 1970s. renewed interest with discovery of chiral *p*-wave superconductor **Sr_2RuO_4** in 1996 and **topological quantum computation/superconductivity**.

- **Raises imp't questions about applicability of mean-field BCS theory to topological superconductivity.**

- Problem can definitively be answered ***experimentally*** by *p*-wave superfluid in harmonic trap.

Angular momentum problem

- Chiral $\Delta(\mathbf{p}) = \Delta(p_x + ip_y)$ superconductor (*A*-phase ^{3}He) breaks time-reversal symmetry: Cooper pairs with **nonzero *L_z* (magnetic) momentum**.



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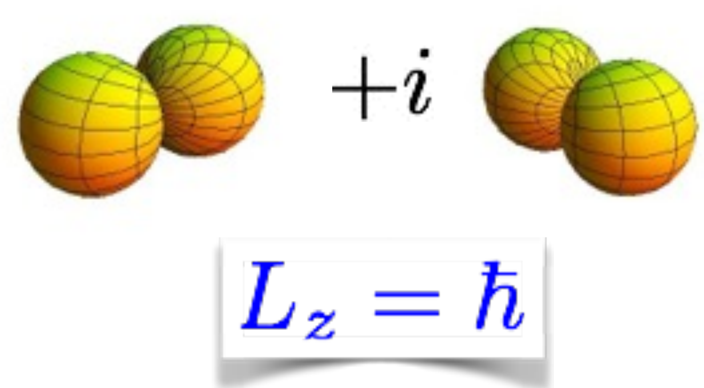

- What is the total angular momentum of a disk of *N* fermions*?

*Cross '75, Leggett & Takagi '78, Combescot '78, Volovik & Mineev, '81, Kita '98, Stone & Roy '04….

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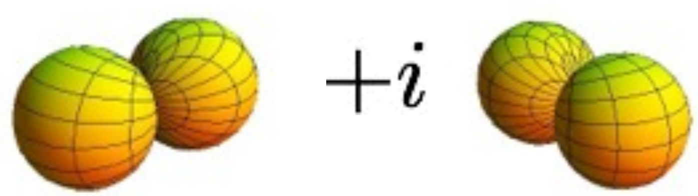

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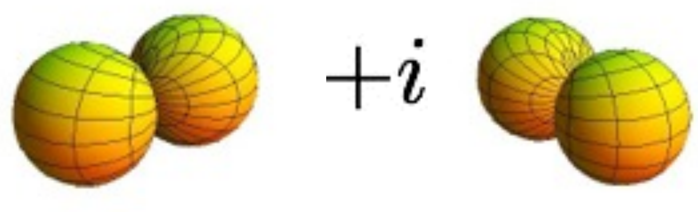

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$$L_z = \frac{N\hbar}{2} \quad \text{or} \quad L_z = \frac{N\hbar}{2}\left(\frac{\Delta}{E_F}\right)^2 \quad ?$$

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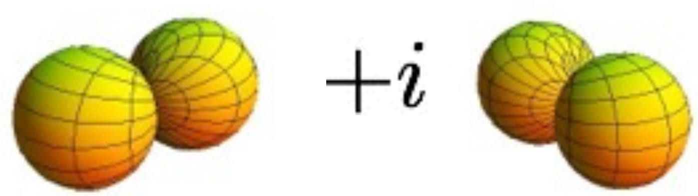

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- ***BCS theory is ambivalent regarding the answer.***

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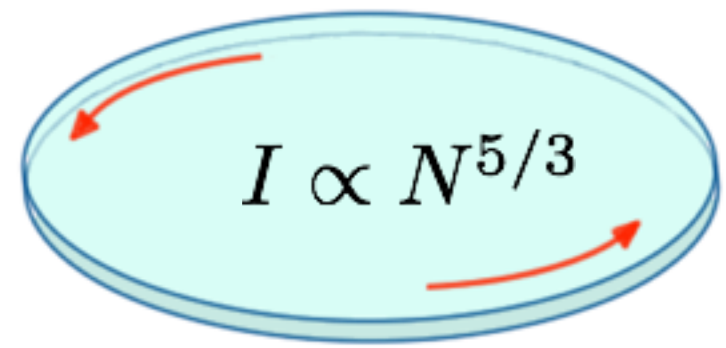
Quasi-degeneracy of edge features in a topological superconductor



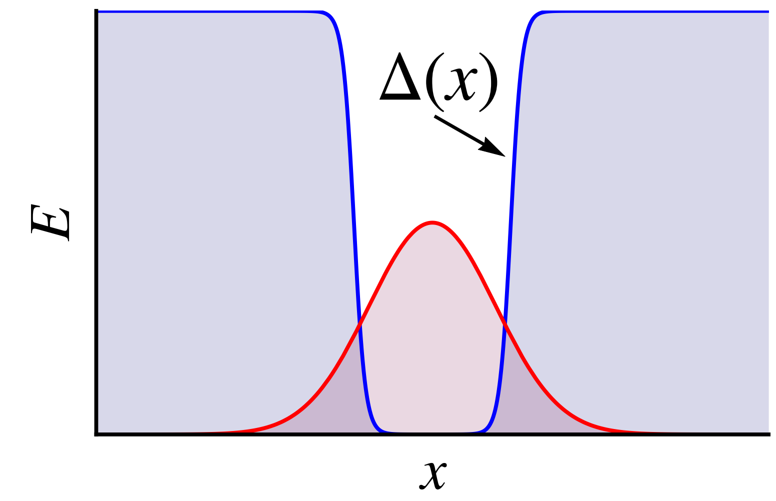
$E = AN^2$

$E = AN^2 + B\Omega N^{5/3}$

- Energy difference ~ $N^{1/3}$. margin of error in BCS: ~ $N^{1/2}$.
- Angular momentum due to edge current carried in part by *Majorana edge states.*
- **Edge features (e.g. *Majorana bound states*) are "small" corrections to the bulk energy. (BCS reliable?)**

Topological superconductors

- *s*, *d*-wave superconductors have (Andreev) edge states with non-zero energy



- "Dirac" dispersion of *p*-wave quasiparticles ⇒ zero-energy edge states: **Majorana bound states**.

$$\hat{\gamma}^\dagger(E) = \hat{\gamma}(-E) \Rightarrow \hat{\gamma}^\dagger(0) = \hat{\gamma}(0)$$

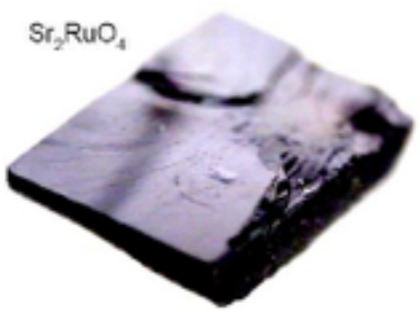
- MBSs act as own antiparticles; don't obey Fermi/Bose statistics; "fractional statistics"⇒ topologically protected (quant. comp.).

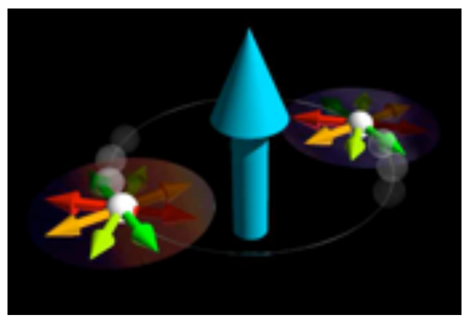
- Topological invariant = Chern number *C*; counts # edge states.

$$C = \frac{1}{4\pi}\int d^k\hat{h}\cdot\left(\partial_{k_x}\hat{h}\times\partial_{k_y}\hat{h}\right);\ \vec{h} \equiv [\mathrm{Re}\Delta(\mathbf{k}), \mathrm{Im}\Delta(\mathbf{k}), \xi(\mathbf{k})]$$

Majorana bound states: candidate systems

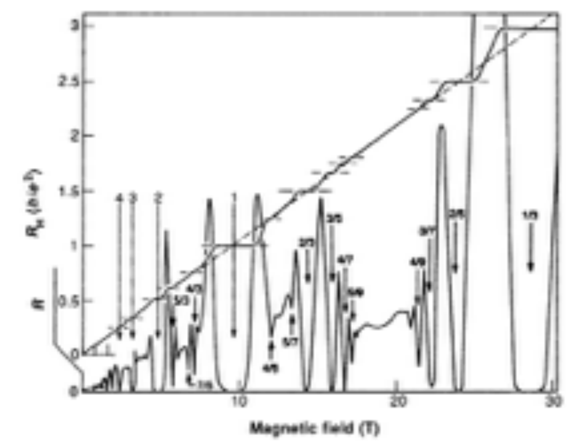
Topological superconductors





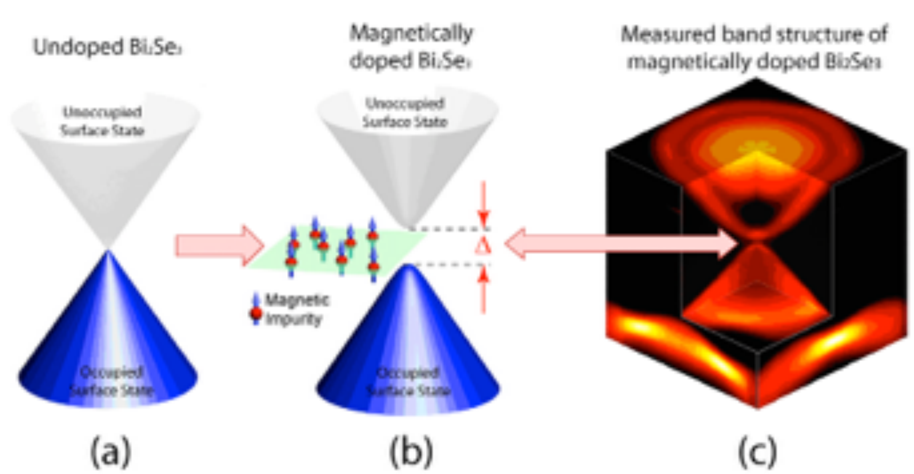
Sr_2RuO_4 (?)

Fractional quantum Hall effect (1982)



GaAs

Topological insulators



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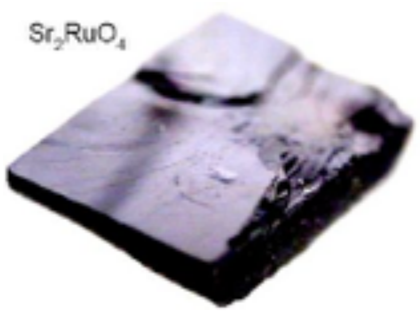

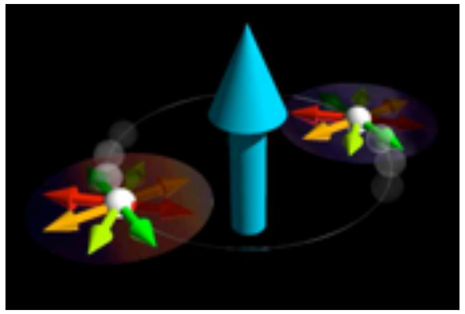

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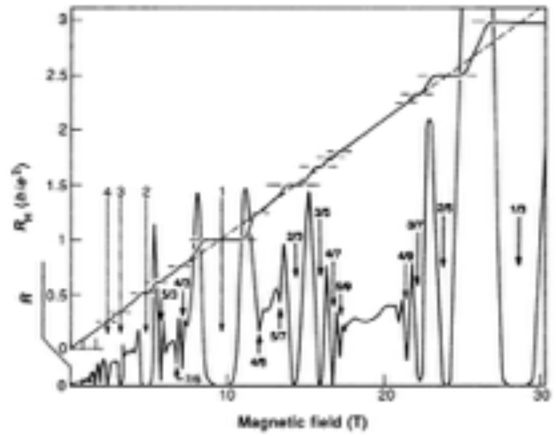

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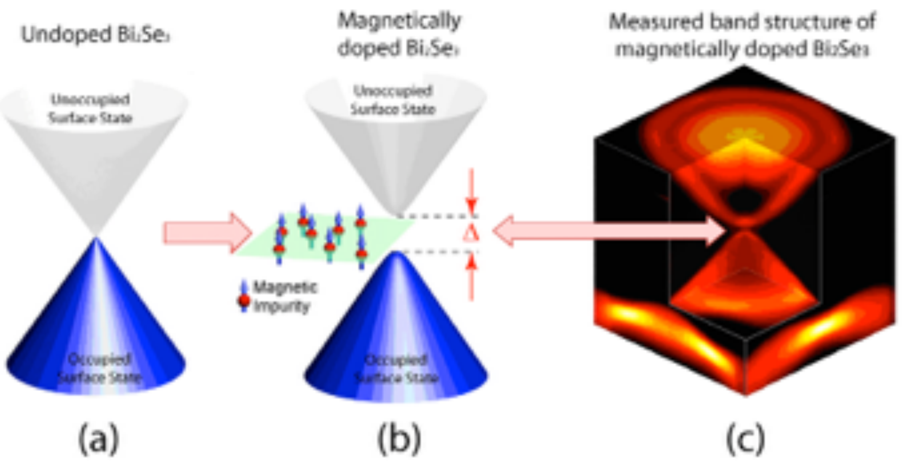

quantized current: $j_y = \frac{e^2}{2\pi} C$

quantized spin current $\propto C$.

Majorana bound states: candidate systems

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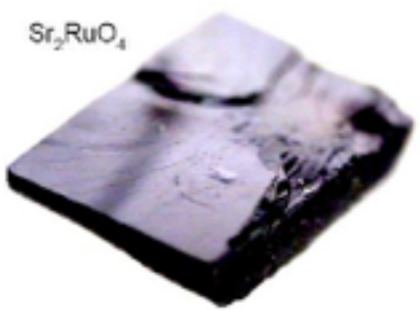

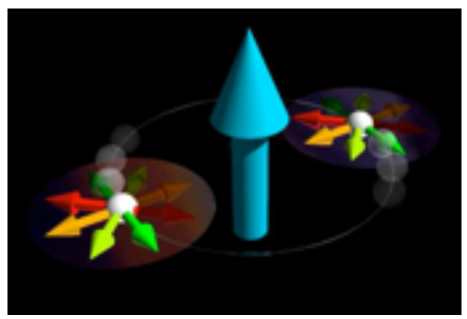

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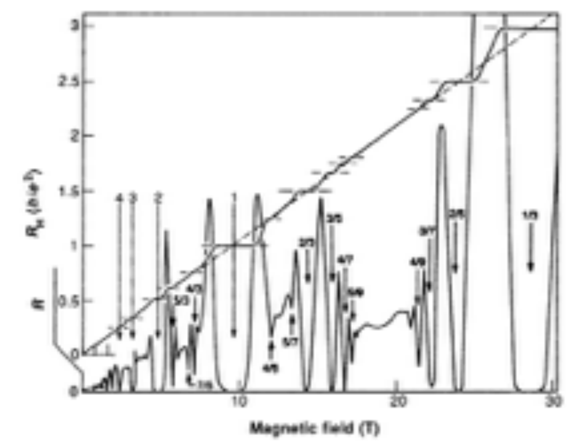

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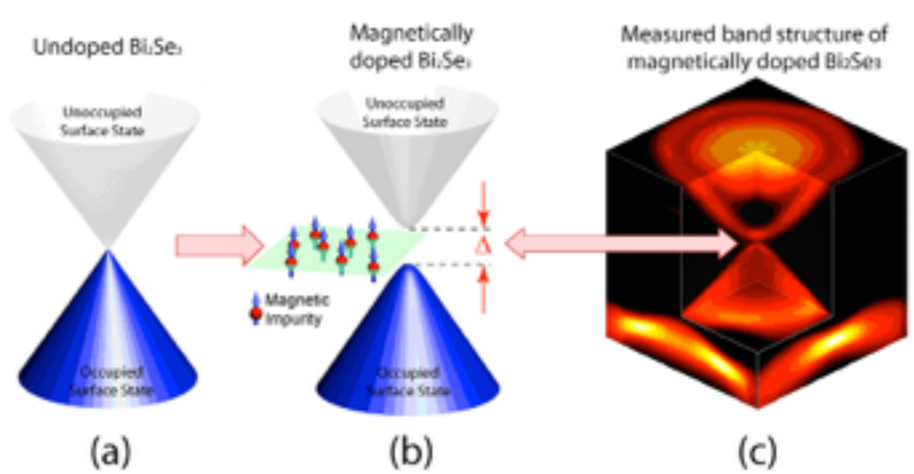


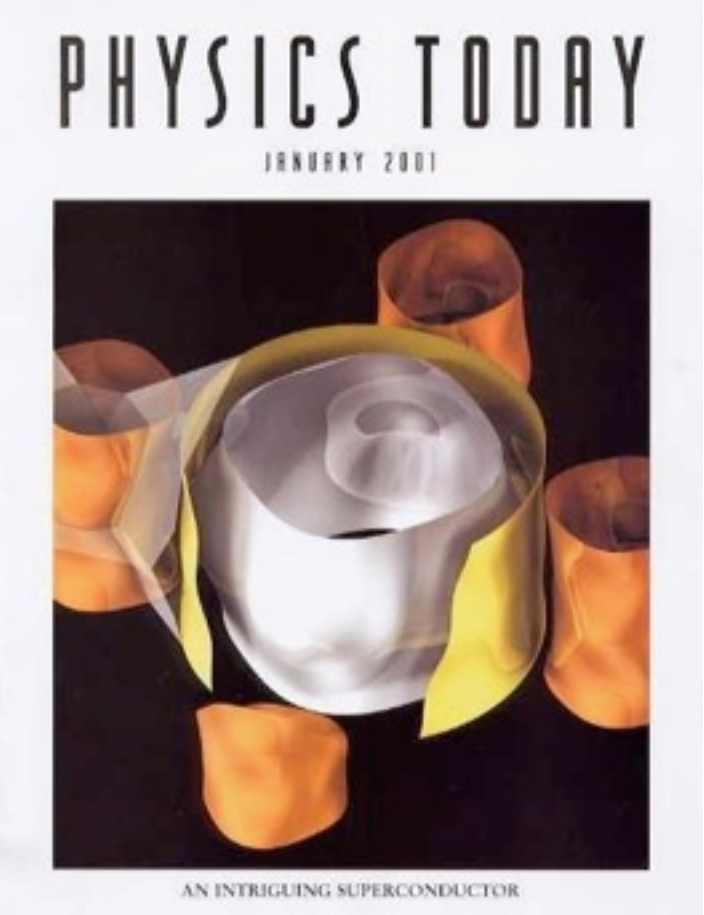
quantized spin current $\propto C$.

- Topological current in chiral *p*-wave SCs? (⇒*large angular momentum*).

Why the angular momentum problem is important

- Topological edge properties of FQH are well established.
- In topological SCs, predictions based on mean-field BCS theory. Existence of MBSs likely well established (Kitaev), but properties (e.g., edge current, braiding/topol. quant. comp.) may not be.
- Adding fuel to the fire: Likely electronic analogue of ^3He discovered in 1996, Sr_2RuO_4. **No signs of an edge current!**

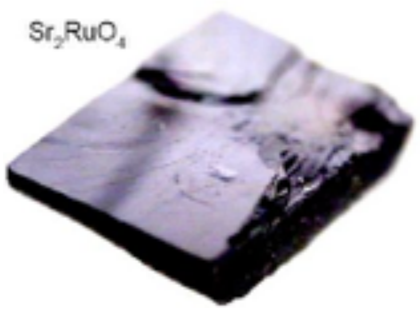
$$10^6 question: Is mean-field BCS reliable when it comes to properties of topological superconductors?



This talk

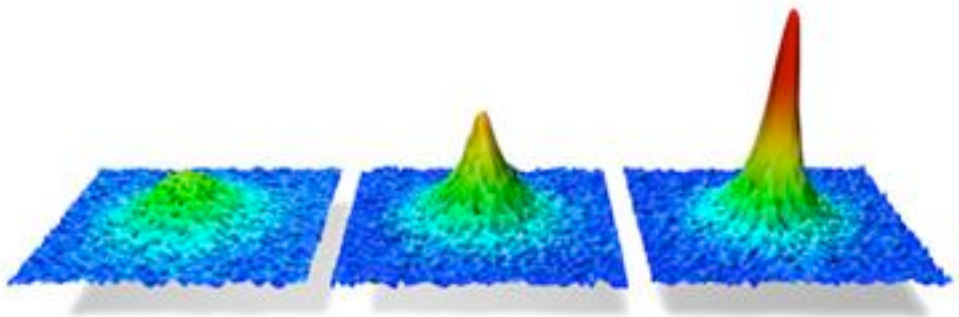
Using standard BCS theory (equiv. $L_z = N\hbar/2$):

- *Edge current sensitive to geometry.* A robust, topological current (related to Majorana states) is only possible if the density varies *slowly* at the edge.



Y. Liu, Penn State

Density varies over atomic scales at edge. Non-topological edge current. *Can* be zero within BCS (e.g., with disorder).



M. Greiner, Harvard

Density varies over long distance. Large, topological edge current within BCS.

In contrast to Sr_2RuO_4, *within BCS*, the edge current in a trapped chiral *p*-wave superfluid is sensitive to the Chern number and is a sensitive test of *mean-field* predictions.

Gradient expansion of the BCS action

$$S_{\text{eff}} = \int dx \int dx' \frac{\Delta(x,x')}{V(x-x')} - \operatorname{Tr} \ln \mathbf{G}^{-1}(x,x')$$

- Minimal coupling to EM field: $\nabla \to \nabla + 2i\mathbf{A};\ \partial_t \to \partial_t - 2iA_0$

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- Expand in powers of slow time and space variations of A:

"Chern-Simons" term

$$\mathcal{L}_{\text{eff}} = \frac{n_{s,\mu\nu}}{2m} A_\mu A_\nu - \frac{n}{2mc^2} A_0^2 - \frac{C}{4\pi} \epsilon_{0\mu\nu} A_0 \partial_\mu A_\nu + \cdots$$

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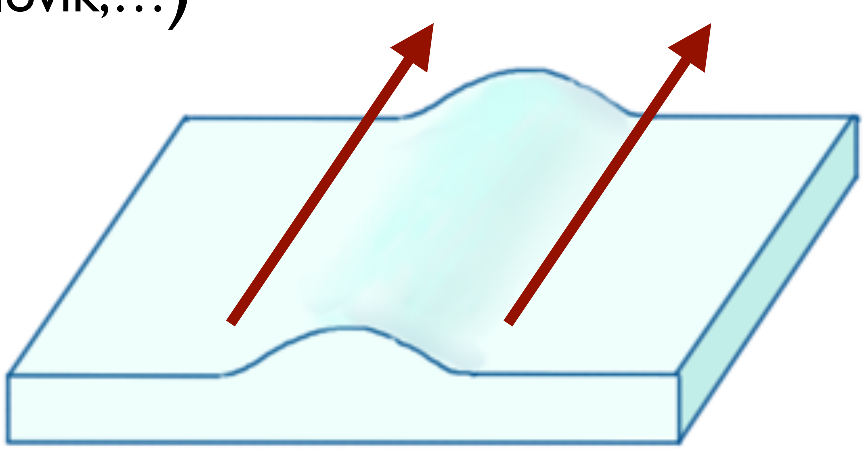
  - "quantum Hall effect" (Ishikawa, Volovik,…)



$$J_y = (\hbar C/4)\partial_x n(x)$$

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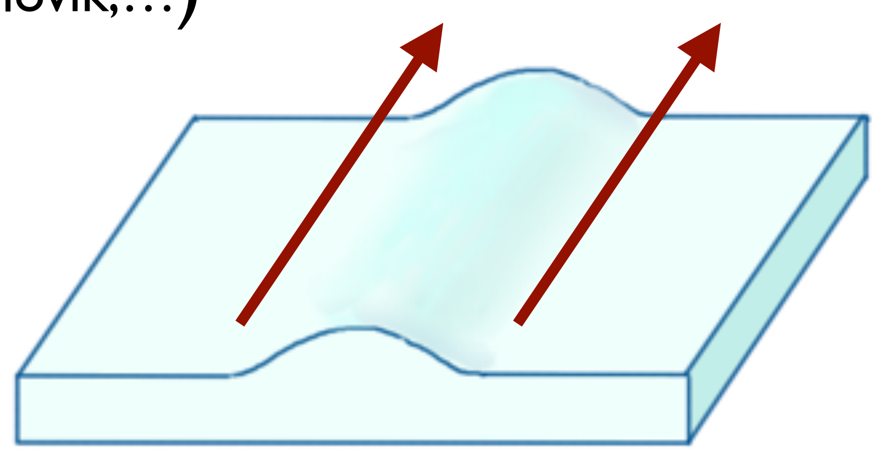
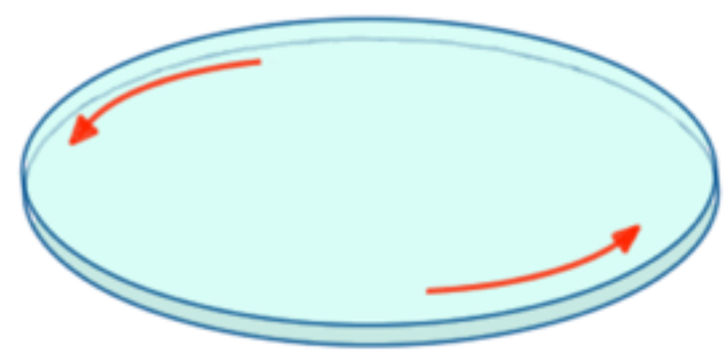


$$J_y = (\hbar C/4) \partial_x n(x)$$

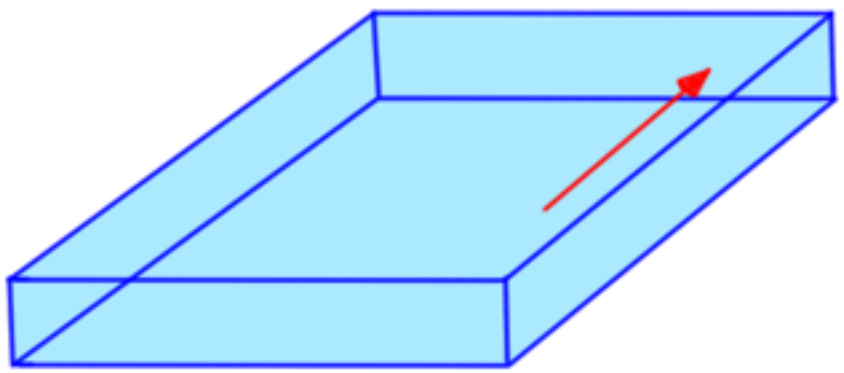
- topological angular momentum



$$L_z = (\hbar C/4) \int d^2 r \partial_r n(r) = C\hbar N/2$$

Edge current

- Apply "topological result" to standard edge problem (e.g., Sr_2RuO_4) where density varies *abruptly* (violates gradient expansion):



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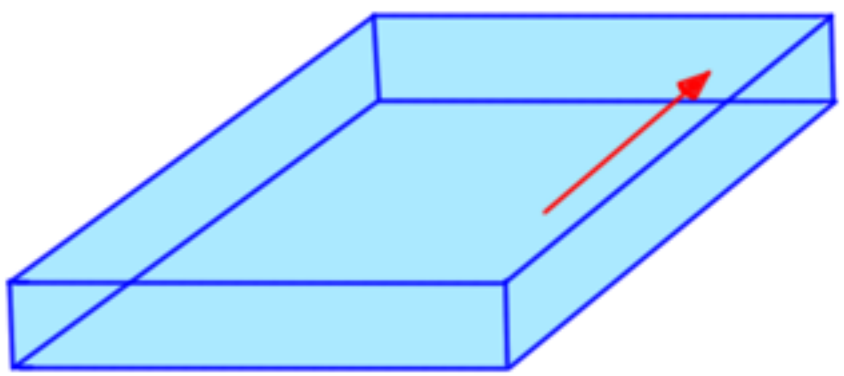
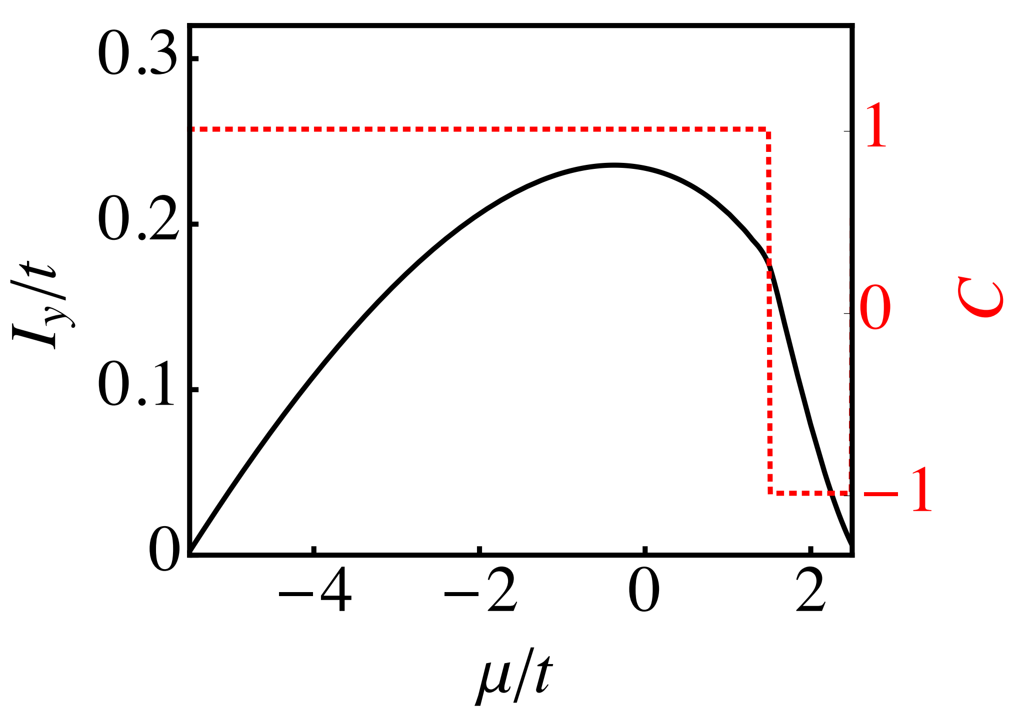


$$I_y = \frac{\hbar C}{4}\int_{-\infty}^{0}\partial_x n(x) = \frac{\hbar C N}{4}$$

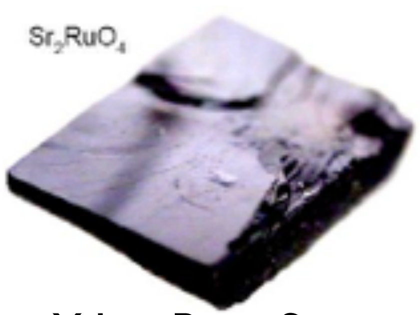
- In contrast, BdG for a lattice model with next-nearest-neighbour hopping:



No correlation with the Chern number. Need non-topological gradient corrections to describe BdG

Geometry matters

- **Hard edge**: Non-topological current. Sensitive to e.g., disorder; can be very small in e.g., Sr_2RuO_4*



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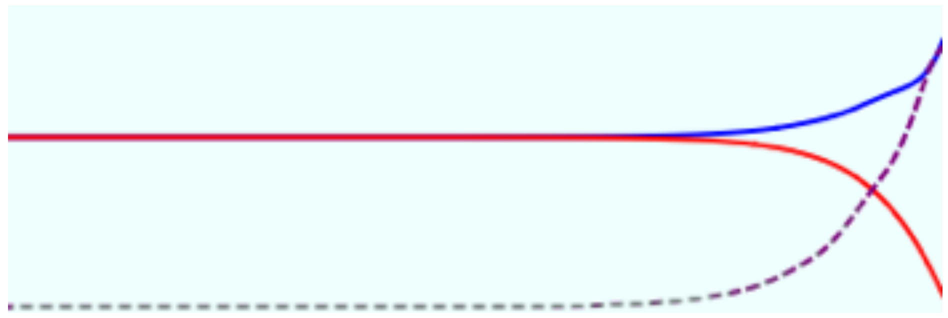
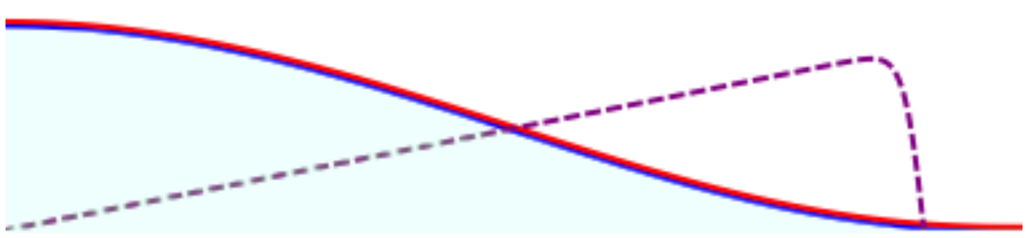


- **Soft edge**: Quasi-topological current; robustly large.


S. Jochim, Heidelberg



$$J_y \simeq \frac{\hbar C}{4} \partial_x n$$

Use cold atoms to probe *topological* edge properties.

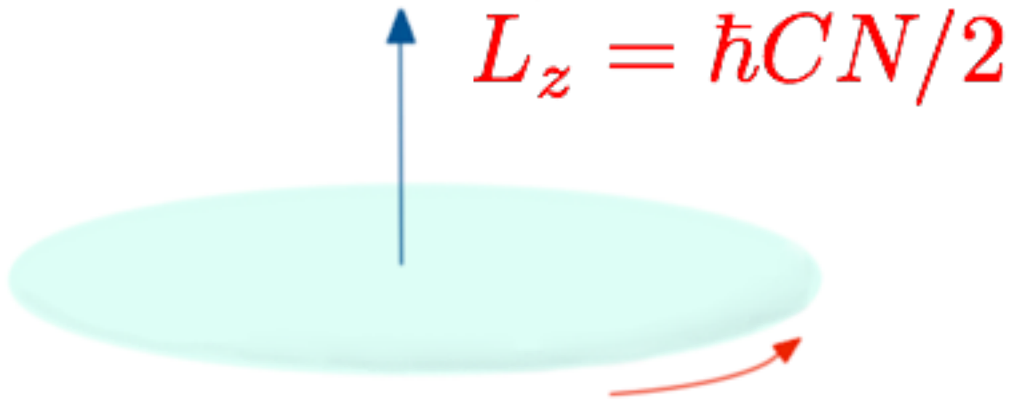
* "*Suppression of spontaneous currents in Sr_2RuO_4 due to surface disorder*", S. Lederer, W. Huang, ET, S. Raghu, & C. Kallin; arXiv:1404.4637

Trapped chiral *p*-wave atomic gas superfluids

- Several proposals for realizing chiral *p*-wave gases:

  Zhang, Tewari, Lutchyn, Das Sarma PRL '08, Levinson, Cooper & Shlyapnikov PRA '11, Juliá-Díaz, Graß, Dutta, Chang, Lewenstein Nature '13, Buhler *et al.*, arXiv:1403.0593…

- Topological angular momentum of a quasi-2D harmonically confined chiral *p*-wave superfluid:



- Angular momentum can be measured by e.g., splitting of quadrupole mode frequency (Zambelli & Stringari, PRL '98, Chevy, Madison, and Dalibard, '00).

Angular momentum in ^{3}He: What happens if the Cooper pairs are molecules?

Diatomic Molecules and Cooper Pairs

by

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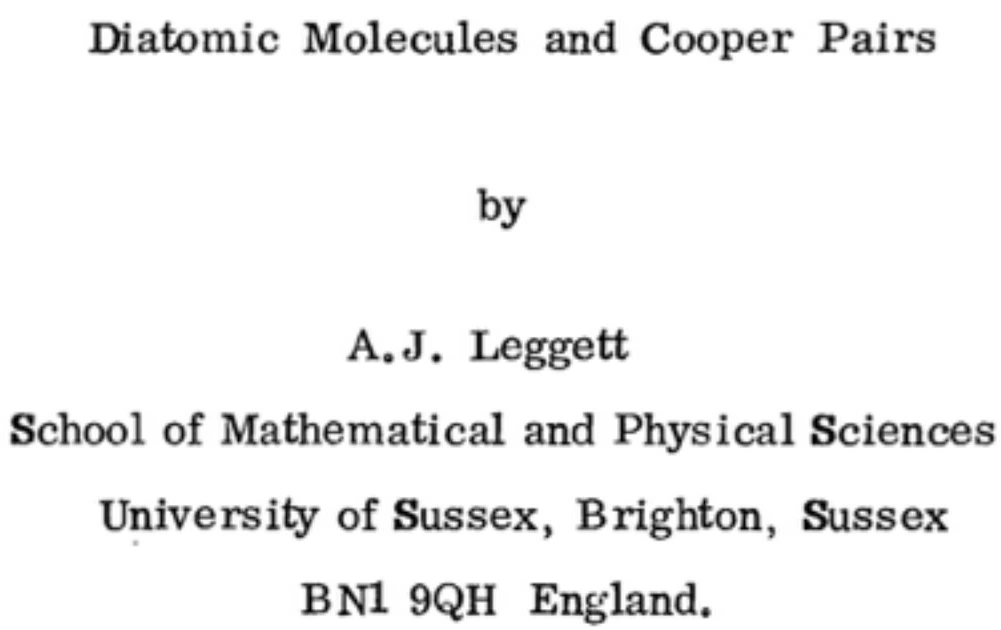
University of Sussex, Brighton, Sussex
BN1 9QH England.

can be polarized etc. All these features suggest that one may gain qualitative insights into the likely behaviour of superfluid ^{3}He by regarding the pairs as like diatomic molecules.

- BCS-BEC crossover: thought experiment to understand angular momentum in ^{3}He-*A*.

BCS-BEC crossover



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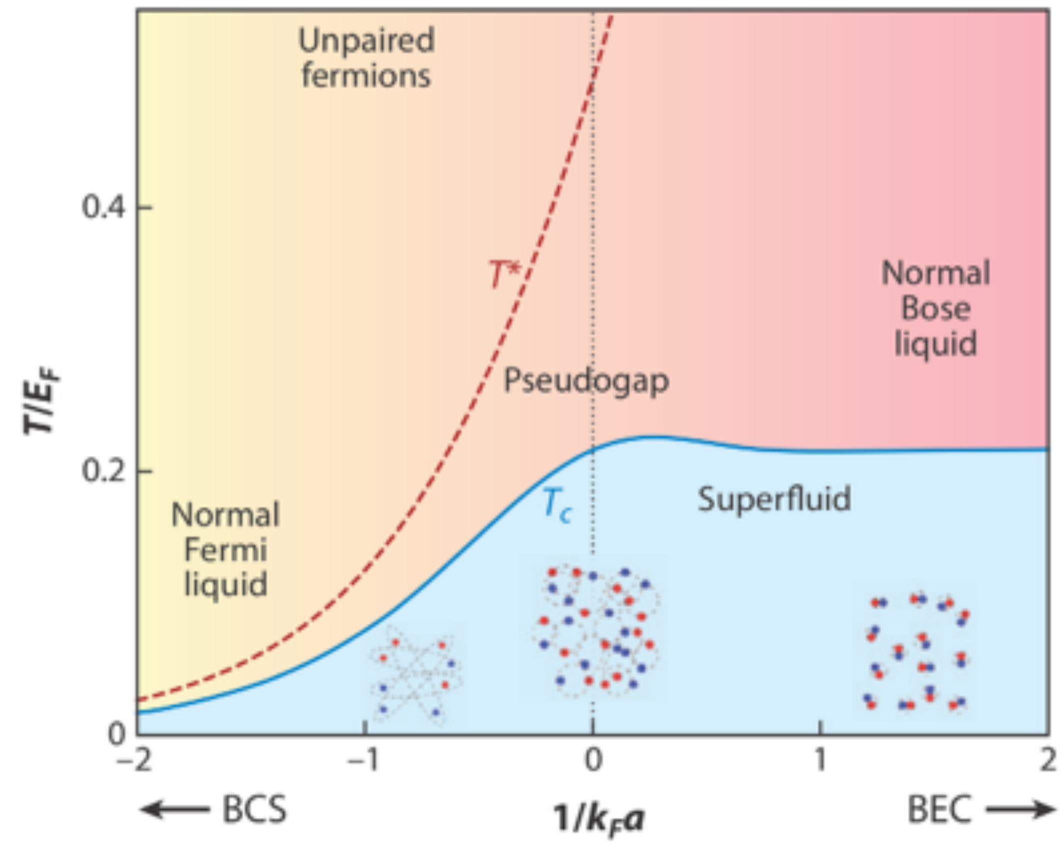
AJ Leggett, In Modern Trends in the Theory of Condensed Matter. Lecture Notes in Physics 115, 13 (1980).



Mohit Randeria & ET, Annual Rev. Condens. Matter Phys. 2014.

- Paradigm for ultra-cold Fermi gas experiments.

The central question posed by the BCS-BEC crossover — what is the angular momentum of ^3He — remains unanswered. Cold atoms can do this, and more (Chern!).

Summary

- 40 year-old problem: What is the angular momentum of a chiral *p*-wave superfluid?

- Relevancy for topological superconductivity: Does mean-field BCS give a good description of the properties of edge states?

- Robust topological edge current ($\propto$# Majorana bound states) only when the density varies slowly near the edge. Probably only a chiral *p*-wave cold atom experiment can probe this.

Thank you!

Work on Sr_2RuO_4 done with Wen Huang, Sam Lederer, Sri Raghu, and Catherine Kallin.

Chern-Simons theory of FQHE

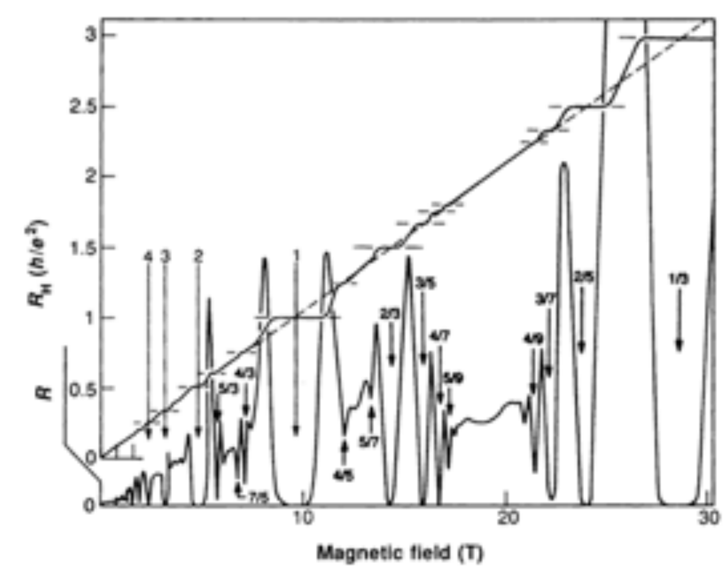
- Girvin & MacDonald '98; Zhang, Hansson, & Kivelson '89:

$$\mathcal{L} = -\frac{e^2}{4\pi} C \epsilon^{\mu\nu\sigma} A_\mu \partial_\nu A_\sigma + \text{stuff}$$

- Topological current prop. to Chern number (=filling fraction).

$$j_\mu = \frac{\delta \mathcal{L}}{\delta A_\mu} \Rightarrow j_y = \frac{e^2}{2\pi} C E_x; \; C = 1, 2, ..., \frac{1}{3}, \frac{5}{2}, ...$$

Robust, topologically protected edge current; insensitive to dirt.



- Suggests topol. current in a chiral *p*-wave superconductors and hence, *large angular momentum*.