

Conference ‘Building(s) in Aarhus’: titles and abstracts

Organizer: Corina Ciobotaru*

August 30, 2022

1. Shaked Bader, Technion - Israel Institute of Technology, Israel

Title: Rank 2 affine buildings are 2-medians

Abstract: A (1-)median space is a space in which for every three points the intersection of the three intervals between them is a unique point. Having this in mind, in my talk I will define a 2-median space which will be a 2 dimensional variation of the median space. I will then present some ideas in the proof of the theorem in the title. This talk is based on my Master Thesis done under the supervision of Nir Lazarovich.

2. Uri Bader, Weizmann Institute of Science, Israel

Title: Dynamics on spaces associated to affine buildings and applications

Abstract: I will explain how to construct various spaces associated with a group action on an affine building and to endow these spaces with natural measures. Sometimes these spaces reveal some hidden symmetries of the building and they allow using techniques from topological dynamics and ergodic theory. The talk is based on a joint work with Jean Lecureux.

3. Benjamin Brück, ETH Zürich, Switzerland

Title: High-dimensional rational cohomology of $\mathrm{SL}_n(\mathbb{Z})$ and $\mathrm{Sp}_{2n}(\mathbb{Z})$

Abstract: By a result of Church–Putman, the rational cohomology of $\mathrm{SL}_n(\mathbb{Z})$ vanishes in ‘codimension one’, i.e. $H^{\binom{n}{2}-1}(\mathrm{SL}_n(\mathbb{Z}); \mathbb{Q}) = 0$ for $n \geq 3$, where $\binom{n}{2}$ is the virtual cohomological dimension of $\mathrm{SL}_n(\mathbb{Z})$. I will talk about two generalisations of this result:

The first project is joint work with Miller–Patz–Sroka–Wilson (arxiv.org/abs/2204.11967). We show that the rational cohomology of $\mathrm{SL}_n(\mathbb{Z})$ vanishes in codimension two, i.e. $H^{\binom{n}{2}-2}(\mathrm{SL}_n(\mathbb{Z}); \mathbb{Q}) = 0$ for $n \geq 3$. The second project is work in progress with Patz–Sroka. Its aim is to study whether the rational cohomology of the symplectic group $\mathrm{Sp}_{2n}(\mathbb{Z})$ vanishes in codimension one, i.e. whether $H^{n^2-1}(\mathrm{Sp}_{2n}(\mathbb{Z}); \mathbb{Q}) = 0$ for $n \geq 2$.

The key ingredient for these results is that we obtain explicit partial resolutions of Steinberg modules, i.e. the top-dimensional homology groups of associated spherical buildings.

4. Jens Niklas Eberhardt, University of Wuppertal, Germany (Online)

Title: A K-theoretic approach to geometric representation theory

Abstract: Perverse sheaves and intersection cohomology are central objects in geometric representation theory. This talk is about their long-lost K-theoretic cousins, called K-motives. We will discuss definitions and basic properties of K-motives and explore potential applications to geometric representation theory. For example, K-motives shed a new light on Beilinson–Ginzburg–Soergel’s Koszul duality – a remarkable symmetry in the representation theory and geometry of two Langlands dual reductive groups. We will see that this new form of Koszul duality does not involve any gradings or mixed geometry which are as essential as mysterious in the classical approaches.

5. Jessica Fintzen, Universität Bonn, Germany

Title: Representations of p -adic groups and the Bruhat–Tits buildings

Abstract: The representation theory of p -adic groups has been revolutionized by the introduction of the Bruhat–Tits buildings. A fundamental problem in the representation theory of p -adic groups is the construction of all (supercuspidal irreducible, smooth, complex) representations of p -adic groups. I will provide an overview of our understanding of the construction of these representations and outline the role played by Bruhat–Tits theory.

*cociobotaru@aias.au.dk

6. Ido Grayeovsky, University of Oxford, U.K.

Title: Sublinear Rigidity of Lattices in Symmetric Spaces

Abstract: Let X be a symmetric space of noncompact type, $\Gamma \leq \text{Isom}(X)$ a lattice and $\Lambda \leq \text{Isom}(X)$ some discrete subgroup. In my PhD thesis, I proved that if Γ is ‘sublinearly covered’ by Λ , then Λ is also a lattice. In case X is of higher rank and Γ non-uniform, the geometry of the associated ‘compact core’ is governed by the rational Tits building structure in the visual boundary of X . This structure turns out to be sublinearly rigid. I plan to present this result within the general context of ‘metric rigidity’, and to present the geometry of the compact core that comes into play in this type of results.

7. Auguste Hébert, Université de Lorraine, France

Title: Λ -buildings associated with quasi-split reductive groups over Λ -valued fields

Abstract: Let K be a field equipped with a valuation $\omega : K \rightarrow \Lambda \cup \{\infty\}$, where Λ is a totally ordered abelian group, not necessarily contained in \mathbb{R} . For example we can take $K = k((x_1))((x_2)) \dots ((x_n))$, where k is a field, x_1, \dots, x_n are indeterminates and $\Lambda = \mathbf{Z}^n$ equipped with the lexicographical order. Let \mathbf{G} be a split or quasi-split reductive group over K (with additional assumptions on K if \mathbf{G} is not split). Let $G = \mathbf{G}(K)$. With Izquierdo and Loisel, we associated to G a ‘ Λ -building’ (in the sense of Bennett) on which G acts. In this talk, I will explain our construction and describe the building that we defined.

8. Linus Kramer, Münster University, Germany

Title: Old and new questions about commutators in compact Lie algebras

Abstract: The structure of compact Lie algebras is a very classical topic which dates back to the fundamental work of Killing, Cartan and Weyl. Nevertheless, there are some questions about the Lie bracket in these algebras which are not easy to answer. We will show that given two elements A, B in a semisimple compact Lie algebra, there always exist elements X, Y, Z with $A = [X, Y]$ and $B = [X, Z]$. If a similar result could be shown for compact simple Lie groups, this would have implications for the Friedlander–Milnor conjecture in dimension 2. On a more basic level, we will see that there are open problems even for the Lie algebra $\mathfrak{su}(6)$.

9. Arielle Leitner, Afeka College of Engineering, Israel

Title: Chabauty Limits of Subgroups in $\text{SL}(n, \mathbb{Q}_p)$

Abstract: Given a second countable locally compact group, G , the set of all subgroups $\mathcal{S}(G)$ may be endowed with the Chabauty topology, under which it is a compact space. In general, it is difficult to understand the full topology of the space $\mathcal{S}(G)$, and the complete homeomorphism type is known only in very few cases. In the first part of the talk we will give an introduction to the Chabauty topology. Then we will study limits of different kinds of subgroups in $\text{SL}(n, \mathbb{Q}_p)$ using the geometry of the Bruhat–Tits building, and the action of the groups on the building. This will give us insight into parts of $\mathcal{S}(\text{SL}(n, \mathbb{Q}_p))$. Based on several joint projects with Corina Ciobotaru and one that also includes Alain Valette.

10. Mathias Løkkegaard Laursen, Aarhus University, Denmark

Title: A p -adic Duffin–Schaeffer conjecture

Abstract: A very central result in metric Diophantine approximation is the Duffin–Schaeffer Theorem, which was proven in 2020 by Koukoulopoulos and Maynard. Given an approximating function $\psi : \mathbb{N} \rightarrow \mathbb{R}_{\geq 0}$, define \mathcal{A} to be the set of numbers α in the unit interval such that $|\alpha - \frac{p}{q}| \leq \frac{\psi(q)}{q}$ for infinitely many reduced fractions $p/q \in \mathbb{Q}$. The Duffin–Schaeffer Theorem then gives a necessary and sufficient condition for determining the Lebesgue measure of \mathcal{A} , which is always either 0 or 1. In this talk, I will discuss how this theorem generalizes into the p -adic context. Here, the set \mathcal{A} has two different natural generalizations: The first generalization, \mathcal{A}^p , considers \mathcal{A} as the limsup set of a specific sequence of sets \mathcal{A}_n , while the other one, \mathcal{B}^p , generalizes the above definition of \mathcal{A} in a more direct fashion. In terms of the p -adic Haar measure, \mathcal{A}^p can be proven to have a theorem much like the Duffin–Schaeffer theorem for \mathcal{A} and is similarly always of measure 0 or 1, while the spectrum of values for the measure of \mathcal{B}^p , on the other hand, is uncountable and does not yet have a Duffin–Schaeffer-like theorem.

11. Anne Thomas, University of Sydney, Australia (Online)

Title: Chimney retractions, folded galleries and affine flag varieties

Abstract: We determine the relationship between the geometry of retractions and the combinatorics of folded galleries for arbitrary affine buildings, and so provide a unified framework to study orbits in affine flag varieties. We introduce the notion of labeled folded galleries for any affine building X and use these to describe the images and preimages of chimney retractions. When X is the building for a group with an affine Tits system, such as the Bruhat–Tits building for a group over a local field, we can then relate labeled folded galleries and shadows to double coset intersections in affine flag varieties. This is joint work with Elizabeth Milićević and Petra Schwer.

12. Stefan Witzel, Giessen University, Germany (Online)

Title: Euclidean buildings, classical and exotic

Abstract: Interest in euclidean buildings traditionally comes from Bruhat–Tits theory: Bruhat–Tits buildings are the non-Archimedean counterparts of symmetric spaces. However, there are two-dimensional euclidean buildings that are not Bruhat–Tits and there is good reason to be interested in them. I will try to explain one such reason.