



*International Workshop*

# **Kähler and non-Kähler geometry: New developments and interactions**

June 21-23, 2022  
AIAS, Aarhus

## **Speakers**

VESTISLAV APOSTOLOV (Nantes)

GIUSEPPE BARBARO (Rome)

SÉBASTIEN BOUCKSOM (Paris)

QUANG-TUAN DANG (Orsay)

THIBAUT DELCROIX (Montpellier)

RUADHAÍ DERVAN (Cambridge)

ANNA FINO (Torino)

FRANCES KIRWAN (Oxford)

EVELINE LEGENDRE (Toulouse)

LIVIU ORNEA (Bucharest)

ANNAMARIA ORTU (Trieste)

JONAS STELZIG (München)

ADRIANO TOMASSINI (Parma)

## **Organizers**

ALEXANDRA OTIMAN (Aarhus)

ZAKARIAS SJÖSTRÖM DYREFELT (Aarhus)

CRISTIANO SPOTTI (Aarhus)

## **Webpage and registration**

[https://aias.au.dk/events/  
international-workshop-kaehler-and-non-kaehler-geometry](https://aias.au.dk/events/international-workshop-kaehler-and-non-kaehler-geometry)

## **Event supported by**

THE VELUX FOUNDATIONS  
VILJUM FONDEN × VELUX FONDEN



AIAS  
AARHUS INSTITUTE  
OF ADVANCED STUDIES



AARHUS  
UNIVERSITY

# Schedule

## Tuesday, June 21

9.00-9.30: Arrival/Registration in the AIAS hall

9.30-10.30: Vestislav Apostolov  
*A Calabi type problem in generalized Kähler geometry*

10.30-11.00: Coffee break

11.00-12.00: Jonas Stelzig  
*On linear relations between cohomological invariants of compact complex manifolds*

12.00-14.00: Lunch

14.00-15.00: Ruadhaí Dervan  
*Extremal Kähler metrics on blowups*

15.00-15.40: Giuseppe Barbaro  
*Second-Chern-Einstein metrics on 4-dimensional almost-Hermitian manifolds*

15.40-16.10: Coffee break

16.10-17.00: Eveline Legendre  
*K-stability and weighted K-stability for Sasaki manifolds and Kähler cones*

## Wednesday, June 22

9.30-10.30: Sébastien Boucksom  
*Non-Archimedean pluripotential theory: a survey*

10.30-10.50: Coffee break

10.50-11.30: Annamaria Ortu  
*Special Kähler metrics on fibrations*

11.30-12.30: Anna Fino  
*Special Hermitian metrics and suspensions*

12.30: Picture+Lunch

Free afternoon

18.30: Social dinner at **Restaurant “Havnaer” Aarhus Ø** (<https://www.havnaer.dk/>)

**Thursday, June 23**

9.30-10.30: Liviu Ornea

*A Calabi-Yau theorem for Vaisman manifolds*

10:30-11.00: Coffee break

11.00-12.00: Thibaut Delcroix

*Effective conditions of existence of extremal Kähler metrics*

12.00-14.00: Lunch

14.00-15.00: Frances Kirwan

*Moment maps for non-reductive group actions*

15.00-15.40: Quan-Tuan Dang

*Kähler-Einstein metrics on log canonical varieties of general type*

15.40-16.10: Coffee break

16.10-17.00: Adriano Tomassini

*On cohomological and metric properties of almost complex manifolds*

\*All talks will take place in the AIAS Auditorium (first floor)

\*\*All talks are 50 minutes + questions or 30 minutes + questions (short talks)

\*\*\* The lunches will be served in the main hall of AIAS

## Abstracts

**Vestislav Apostolov (Nantes)**

*A Calabi type problem in generalized Kähler geometry*

The notion of a generalized Kähler (GK) structure was introduced in the early 2000's by Hitchin and Gualtieri, with the aim of providing a mathematically rigorous framework of certain nonlinear sigma model theories in physics. Since then, the subject developed rapidly and it is now realized, thanks to the work of Hitchin, Goto, Gualtieri, Bischoff and Zabzine, that GK structures are naturally attached to Kähler manifolds endowed with a holomorphic Poisson structure. Inspired by Calabi's program in Kähler geometry, which aims at finding a "canonical" Kähler metric in a fixed deRham class, I will present in this talk an approach towards a "generalized" version of Calabi's problem, passing through an infinite dimensional moment application formalism and using the Bismut-Ricci flow introduced by Streets and Tian. As an application, we give a complete description - conjectured by D. Joyce in 1999- of the GK structures of symplectic type on the torus  $T^{2n}$ .

---

**Giuseppe Barbaro (Sapienza)**

*Second-Chern-Einstein metrics on 4-dimensional almost-Hermitian manifolds*

I will present a joint work with Prof. Mehdi Lejmi. We study the second-Chern-Einstein in the almost-Hermitian setting. We focus on 4-dimensional compact almost-Hermitian manifolds, observing that under a natural hypothesis the Riemannian dual of the Lee form is a Killing vector field. We use that observation to describe 4-dimensional compact second-Chern-Einstein locally conformally symplectic manifolds. Finally, we study the second-Chern-Einstein problem on unimodular almost-abelian Lie algebras, giving a classification of those that admit a left-invariant second-Chern-Einstein metric with a parallel non-zero Lee form.

---

**Sébastien Boucksom (École Polytechnique)**

*Non-Archimedean pluripotential theory: a survey*

The non-Archimedean version of pluripotential theory takes place on analytic spaces in the sense of Berkovich, which are basically spaces of valuations, and studies psh functions and Monge-Ampère equations thereon. The case of a trivially valued ground field is of special interest, due to its close relationship to K-stability, and can be viewed as a far-reaching generalization of the study of real Monge-Ampère equations for convex functions (which correspond to the toric case). The purpose of this talk is to survey some aspects of this theory, and their implications in the study of K-stability.

---

**Quang-Tuan Dang (Orsay)**

*Kähler-Einstein metrics on log canonical varieties of general type*

In this talk we introduce the notion of singular Kähler-Einstein metrics on mildly singular varieties. We prove that solutions of degenerate complex Monge-Ampère equations on compact Kähler manifolds are continuous on a Zariski open set. This allows us to show that singular Kähler-Einstein metrics on log canonical varieties of general type have continuous potentials on the ample locus outside of the non-plt part.

---

---

**Thibaut Delcroix (Montpellier)**

*Effective conditions of existence of extremal Kähler metrics*

I will present how elementary methods produce simple to check sufficient conditions for existence of cscK or extremal Kähler metrics, for manifolds where a uniform YTD-type conjecture is known to hold and where K-stability can be read off from combinatorial data on a polytope. Such manifolds include spherical manifolds and semisimple principal toric fibrations. This is partly joint work with Simon Jubert.

---

**Ruadhaí Dervan (Cambridge)**

*Extremal Kähler metrics on blowups*

One of the central goals of complex geometry is to understand the existence of canonical Kähler metrics. Prominent examples are Kähler-Einstein metrics, constant scalar curvature Kähler metrics and extremal metrics. An important early construction of extremal metrics due to Arezzo, Pacard and Singer demonstrated a beautiful link with algebro-geometric stability in a special geometric setting: namely when one blows up a complex manifold admitting an extremal metric. Their work gave a sufficient criterion under which the blowup admits extremal metrics. I will describe sharp results in this direction, using a new approach. This is joint work with Lars Sektnan.

---

**Anna Fino (Torino)**

*Special Hermitian metrics and suspensions*

In the talk I will report some general results on existence of special Hermitian structures, like balanced, SKT and generalized Kähler, on suspensions. In particular, I will show some recent results on compact solvmanifolds, in collaboration with Fabio Paradiso, and on toric suspensions of hyperKähler manifolds, in collaboration with Gueo Grantcharov and Misha Verbitsky.

---

**Frances Kirwan (Oxford)**

*Moment maps for non-reductive group actions*

Moment maps have been used since the 1970s to study the geometry and topology of GIT quotients by reductive group actions. The aim of this talk is to describe some joint work with Greg Berczi <https://arxiv.org/abs/1909.11495> extending the moment map description of reductive GIT quotients to give an analogous description of GIT quotients by suitable non-reductive group actions.

---

**Eveline Legendre (Toulouse)**

*K-stability and weighted K-stability for Sasaki manifolds and Kähler cones*

We will see how to use weighted versions of the classical Kähler functionals to study Sasaki extremal metrics. As an application we improve to K-stability a result of Collins-Szekelyhidi on K-semistability of cscS manifolds. This is a joint work with V. Apostolov and D.J.M. Calderbank.

---

**Liviu Ornea (Bucharest)**

*A Calabi-Yau theorem for Vaisman manifolds*

A Vaisman manifold is endowed with a transversally Kähler foliation. We construct a correspondence between the set of Vaisman metrics on  $M$  and the set of transversal Kähler structures. Then we show that the transversal Calabi-Yau theorem implies that the Vaisman metric is defined uniquely, up to a constant multiplier, by the volume and the Lee class  $[\theta] \in H^1(M)$ . Since the space of possible Lee classes on  $M$  is identified with a certain open half-space in  $H^1(M)$ , it follows that the set of all Vaisman structures on a compact complex manifold  $(M, I)$  is parametrized by the cohomological data together with the volumes. This is a joint work with Misha Verbitsky.

---

**Annamaria Ortu (SISSA Trieste)**

*Special Kähler metrics on fibrations*

Proper holomorphic submersions of Kähler manifolds can be thought of as both a generalisation of holomorphic vector bundles and as a way of studying the behaviour of Kähler manifolds in families. We will consider fibrations whose fibres are K-semistable deformations of Kähler manifolds with constant scalar curvature, in a way compatible with the fibration structure. On such fibrations, we will describe a canonical choice of a relatively Kähler metric, called an optimal symplectic connection, that provides a generalisation of the Hermite-Einstein condition on vector bundles and gives the existence of special Kähler metrics on the total space.

---

**Jonas Stelzig (LMU München)**

*On linear relations between cohomological invariants of compact complex manifolds*

In his problem list from the 50s, Hirzebruch asked which linear combinations of Hodge and Chern numbers are topological invariants of compact projective, Kähler, or even all complex manifolds. The projective/Kähler case was solved by Kotschick and Schreider. Further developing their ideas, we present a solution of the general complex manifold case. We also point out a more general question and reduce its answer to a finite number of construction problems in every dimension.

---

**Adriano Tomassini (Parma)**

*On cohomological and metric properties of almost complex manifolds*

---