EFIMOV TRIMERS
UNDER STRONG CONFINEMENT

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STRONG CONFINEMENT EFFECTS

The dimensionality of the embedding space profoundly affects the system properties.

Examples:

- Anderson localization
- condensation & superfluidity
OUTLINE

- Three identical bosons: 3D vs. 2D
- What happens in between? (quasi-2D)
  - trimer spectra and aspect-ratios
  - hyper-spherical potentials and wave functions
- Experimental consequences
2&3 IDENTICAL BOSONS IN 3D

One universal dimer: \( E_b = -\frac{\hbar^2}{ma^2} \quad (a > 0) \)

For resonant interactions (\( 1/a = 0 \)), in principle \( \exists \) an infinite tower of Efimov trimers.

Trimers map onto each other via the scale transformations \( a \to \lambda_0^n a \) and \( E \to \lambda_0^{-2n} E \)

\[ \text{Smallest set by short-distance physics} \]

\[ \text{Scaling symmetry: continuous (two-body) vs. discrete (three-body)} \]
**2&3 IDENTICAL BOSONS IN 2D**

Apply harmonic confinement: 
\[ V(z) = \frac{1}{2} m \omega_z^2 z^2 \]

- CoM decouples
- continuum is shifted
- additional length scale appears

**One universal dimer:** 
\[ E_b = \frac{\hbar \omega}{2} - \frac{\hbar^2}{ma_{2D}^2} \]

Petrov & Shlyapnikov PRA 2001  
Bloch, Dalibard, Zwerger RMP 2008

**Two universal trimers:**

-1.27 \[ \frac{\hbar^2}{ma_{2D}^2} \]

-16.5 \[ \frac{\hbar^2}{ma_{2D}^2} \]

Bruch & Tjon, PRA 1979

Both two- and three-body problems display a continuous scaling symmetry

2D Bose gas experiments: Paris, Innsbruck, Chicago, Monash, …
THREE BOSONS IN QUASI-2D

\[ H = \sum_{k,n} (\epsilon_k + n\hbar \omega_z) a_{k,n}^\dagger a_{k,n} + \sum_{k',q} \frac{e^{-(k^2+k'^2)/\Lambda^2}}{\Lambda^2} \langle n_1 n_2 | \hat{g} | n_3 n_4 \rangle a_{q/2+k,n_1}^\dagger a_{q/2-k,n_2} a_{q/2-k',n_3} a_{q/2+k',n_4} \]

the UV cut-off \( \Lambda \) controls the three-body physics at short-distances, and fixes the crossing of the deepest Efimov trimer with the 3-atom continuum (a.)

Trimer wave function: \[ \sum_{k_1,k_2} \psi_{k_1,k_2}^{n_1,n_2,n_3} a_{k_1,n_1}^\dagger a_{k_2,n_2}^\dagger a_{-k_1-k_2,n_3} \]


see also the recent work on trimers in a box by M.Yamashita et al., arXiv:1404.7002
atom-dimer vertex:

\[ \mathcal{T}^{-1} (k_1, E_3 - \epsilon_{k_1} - N_1\omega_z) \chi_{k_1}^{N_1} = 2 \sum_{k_2, n_{23}, n_{31}} \frac{f_{n_{23}} f_{n_{31}} \langle N_1 n_{23} | N_2 n_{31} \rangle e^{-(k_1^2 + k_2^2)/\Lambda^2}}{E_3 - \epsilon_{k_1} - \epsilon_{k_2} - \epsilon_{k_1+k_2} - (N_1 + n_{23})\omega_z} \chi_{k_2}^{N_2} \]

(the CoM q.number N does not appear in the final formula!)

Wave function for the atom-pair relative motion:

\[ \psi(\rho, Z) = R^{3/2} \sum_{k, N} e^{ik\cdot\rho} \phi_N(Z) \chi_k^N \]

where:
- \( \mathcal{T}(k, E) = \frac{2\sqrt{2\pi}}{m} \left\{ \frac{l_z}{a} - \mathcal{F} \left( \frac{-E + k^2/4m}{\omega_L} \right) \right\}^{-1} \)
- \( E_3 \) is the energy measured from the 3-atom continuum
- \( k_i \) is the relative momentum of atom \( i \) w.r.t. the pair \((j, k)\)
- \( n_{ij} \) and \( N_i \) are the h.o. quantum numbers for motion along \( z \) of a pair, and of an atom and a pair

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SPECTRUM

“weak” confinement

interaction strength: $|a_-|/a$

confinement strength: $C_z \equiv |a_-|/l_z$

strong confinement

$^{133}\text{Cs: } \omega_z \approx 2\pi \times 5\text{kHz}$

$\omega_z \approx 2\pi \times 30\text{kHz}$

- deepest trimer closely resembles the 3D-one, even for strong confinement
- spectrum of trimers is strongly modified above the 3D continuum
- energy of trimer (measured from the q2D dimer) can be a significant fraction of $\omega_z$ even when $|a_-|/a<-1$, so trimers can be quite resistant to thermal dissociation when $T<<\omega_z$
SPECTRUM (2D STYLE)

\[ C_z \equiv |a_-|/l_z \]

- the 2D limit is recovered for small and negative scattering lengths ("BCS side" of the resonance)
- the two deepest trimers are stabilized for every negative scattering length
- avoided crossings: superposition of trimers with Efimovian + 2D-like character

\[ ^{133}\text{Cs}: \quad \omega_z \approx 2\pi \times 5\text{kHz} \]

\[ \omega_z \approx 2\pi \times 30\text{kHz} \]
SHAPE OF THE TRIMERS

2D limit

3D regime
HYPERSPHERICAL POTENTIALS

Hyper-spherical expansion:

\[
\Psi(R, \Omega) = \frac{1}{R^{5/2} \sin(2\alpha_k)} \sum_{n=0}^{\infty} f_n(R) \Phi_n(R, \Omega)
\]

Hyper-radial Schrödinger equation:

\[
\left[-\frac{1}{2m} \frac{\partial^2}{\partial R^2} + V(R)\right] f_0(R) = (E_3 + \omega_z) f_0(R)
\]

\[V(R)\] depends on \(l_z/a\), but not on the 3-body parameter.
HYPERSPHERICAL POTENTIALS

• $V(R)$ approaches the 3D potential for $R \ll |a|$ and the 2D potential for $R \gg l_z$
• When $l_z/a \lesssim -2.5$ the potential displays a repulsive barrier with height $\sim 0.15/ma^2$
• Small weight of trimers in the short distance region enhances lifetime
EXPERIMENTAL CONSEQUENCES

- As “2D” experiments are performed at confinements often weaker than 5kHz, we expect this crossover physics to impact three-body correlations in realistic 2D studies on the attractive side of the Feshbach resonance.

- Confinement raises continuum by \( \hbar \omega_z \), so trimer resonance and loss peak disappear for \( l_z / |a_-| \lesssim 2.5 \), i.e., \( C_z \gtrsim 0.4 \).

- When aiming at observing the discrete scaling symmetry: the 2nd trimer signature disappears once \( C_z \gtrsim 0.4/22.7 \), which for \(^{133}\text{Cs}\) corresponds to \( \omega_z \approx 2\pi \times 10\text{Hz} \).

- Similar effects expected for 4-body states (as two tetrabers exist in 2D), or in quasi-1D.

observation of 2\(^{nd}\) resonance in Cs recently reported by Bo Huang et al, PRL (2014)
Efimov trimers under strong confinement

Discrete scaling survives only for $|a_-| \ll |a| \ll l_z$

Deepest trimer remains 3D-like even under strong confinement

Mixing with 2D trimers stabilizes the two deepest trimers for all $a<0$

Small weight at short distance will enhance lifetime (long-lived Efimov trimers?)

Consequences for correlations, quest to observe discrete scaling symmetry
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